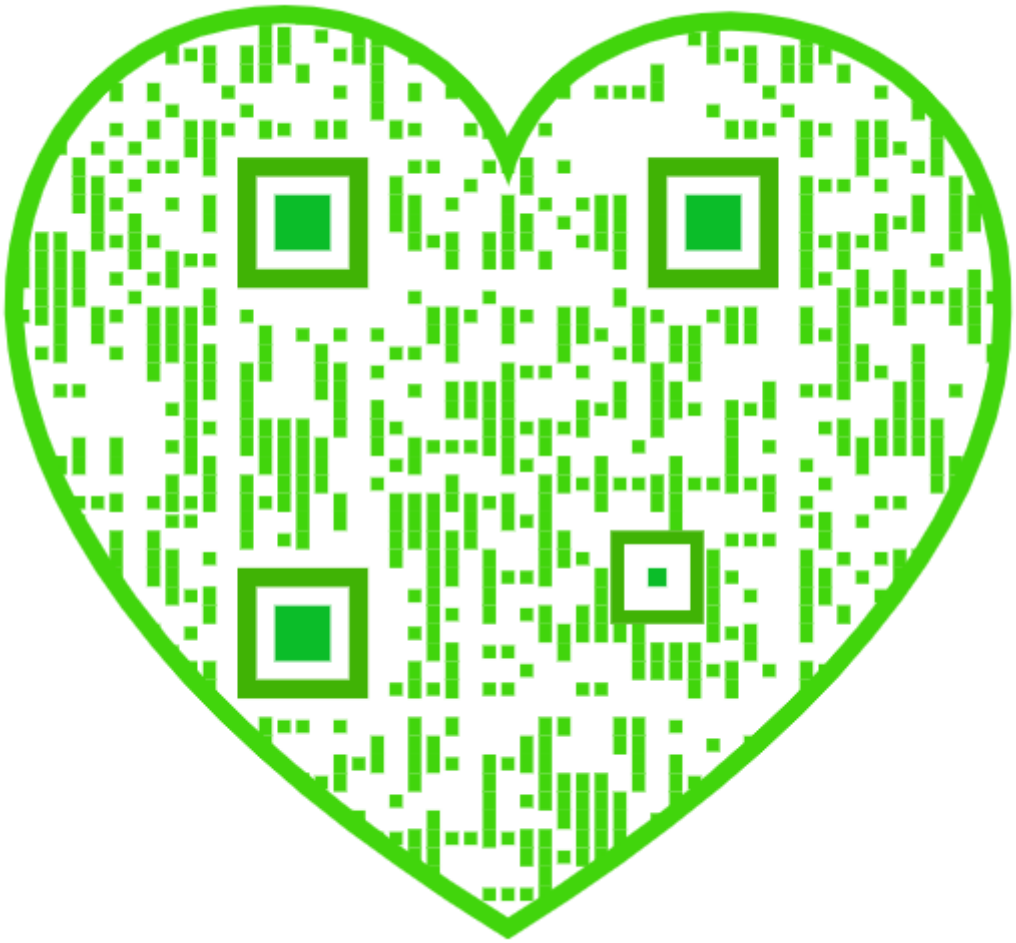

ELEMENTARY

ARITHMETIC

BY

WILLIAM W. SPEER

ASSISTANT SUPERINTENDENT OF SCHOOLS, CHICAGO

BOSTON, U.S.A., AND LONDON

GINN & COMPANY, PUBLISHERS

The Athenæum Press

1897

THE NEW YORK
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INTRODUCTION.

Normal State of the Pupil. If the work of the first year has been rightly carried on there will be a tendency to look at things, to handle them, to compare them, and to express freely what is discovered. Continued growth requires continued fostering of this disposition. The mind that is not exercised in observing and comparing loses power to picture conditions, to see things as they are. Growing power to judge involves continued sensing, imaging, and expressing.

Condition of Learner should shape Work. Whether certain work is, or is not, adapted to children is not to be decided by age or grade, but by their mental condition. If the beginning has opposed the order of the mind's unfolding, there will be corresponding weakness. Growth into any order of truth does not *begin*[1] with definite judgments. A presentation in accord with the forces within permits the student of things in their mathematical aspect to move naturally from

[1] "All the force of Nature could not develop a nerve cell directly out of inorganic matter." — Dr. Maudsley.

"There must be a mental structure capable of grasping the terms of the proposition and the relation alleged between them; and there must be such definite and deliberate mental representation of these terms as makes possible a clear consciousness of the relation." — Herbert Spencer.

the undefined to the defined, from wholes to parts. It does not oppose the natural order of mental action by beginning with divided magnitudes and thus robbing the pupil of his opportunity to analyze. It does not continue with the "how many" and thus deprive the mind of nutrition.

To substitute counting and "number[1] work" for contact with mathematical relations is to check the development of the representative power and interfere with growing definiteness of mind. A seeming combining and separating which merely involves attention to numeral adjectives no more prepare for mathematical inferences than memorizing tables of genera, species, etc., prepare for biological inferences. Such work obscures the subject, dulls the senses, and leaves the mind without a basis for mathematical judgment. In short, its tendency is to unfit for intelligent action.

Methods which have made the pupil dependent will not make him independent; methods which have left him destitute of ideas will not bring ideas into his mind. If the pupil is without the mental power, the habits of investigation, or the elementary ideas needed, it may be found profitable to turn to the first book. In this book the basis for work in the other grades is laid, with the conviction that progress is, at all times, most surely promoted by work which calls forth *developing* activity of body and mind. The fundamental act of mathematical reasoning is the relating of objects of sense. If such work has been neglected, it is essential to begin it at once, whatever the age of the class.

[1] See remarks by Clifford, Comte, and others, pages 23–30, Teachers' Book.

Elementary Mathematical Work. The work in elementary mathematics should put the learner into possession of the simple relations by means of which relations beyond the range of perception may be indirectly established. It should cause simple, elementary ideas to be so clearly perceived that the resulting inferences will be natural and necessary. It should induce a creative attitude of mind.

The solution of problems is merely the perception of relations. In no case is such perception generated by formulas,[1] or by mechanical processes for arriving at ready-made results. Formal analyses may hide the lack of thought; the self-active mind is the only force[2] which dispels vagueness and haziness. The child's expression at first may show only the germination of an idea, it may but dimly hint even of this; nevertheless, such expression is a vital process, which should not be smothered by imposing upon it formal statements.

Insight into mathematics does away with ready-made analyses. Such insight shows that mathematical relations are just as real as the relations of seed, root, leaf, flower, and fruit. It causes the teacher to provide for perceptive contact with mathematical realities. The growth and the expression of personal judgment follow naturally upon such contact.

[1] "This clearness of the ready-made is a superficial clearness — a physical, outward, solar clearness, so to speak; but in the absence of a sense for origin and genesis, it is the clearness of the incomprehensible, the clearness of opacity, the clearness of the obscure." — Amiel.

[2] "Man cannot invent a force, he directs it. Science consists in imitating nature." — Balzac.

Purpose and Means. — From beginning to end the purpose is to bring into view the quantitative relations of things.

Mathematical relations, whether we call them integers or fractions, become known by continued activity in judging of relative magnitude. The process is but an extension of that which led the child through imperfectly quantitative judgments to perceptions of exact likeness or equality. Thus, *a* is seen to be greater than

b

a

b, *b* less than *a*, by comparing them. In the same way 2 is seen to be the relation of *a* to *b*, and $\frac{1}{2}$ the relation of *b* to *a*. If *a* and the relation $\frac{1}{2}$ are given, *b* can be imaged; if *b* and the relation 2 are given, *a* can be imaged; but to image *a* or *b* or any missing term is impossible if the judgment by which their relative magnitude is realized has not previously taken place. This truth is of universal application.

By direct perception the relative magnitude of 2 and 3 tons of coal cannot be discovered; until the mind is aware of this relation, it has no basis for judging the relative cost of the coal; but if it is in possession of the relation $\frac{2}{3}$, it readily infers that, if 3 tons cost $\$x$, 2 tons cost $\frac{2}{3}$ of $\$x$. How are we to acquire this basic idea?

The possession of simple relations does not grow out of the expression of the relations, nor out of the obser-

vation of one or two things, but out of many experiences in which the relations are felt. (See Teachers' Book, pages 19, 20, 31.) By exercising ourselves upon things which lie within the range of sense, we obtain the means of discovering the relations of things which lie beyond.

Results may be reached mechanically, but the learner solves problems, advances in mathematics, and thereby develops mental power only by meeting the conditions set forth in the above simple illustrations. The nature of mental action and the conditions of judging emphasize these ideas. Mathematical processes should spring naturally from the needs and purposes they are to serve. They are easily mastered by the active mind.[1]

Proper Environment. — A pupil may be held to one set of blocks in learning simple relations in the mistaken idea that closer analyses and clearer images follow this plan.

The need of clear images in every subject is undoubted. But if in the effort to meet this need the mind is treated as a phonograph, stamped with a few fixed forms from which it cannot dissociate the relation they display, it loses the flexibility which is the condition of free mental action. No method enables one to see as a child what he may see as an adult, but this is no reason for adopting methods fatal to mental unity

[1] "The beginning of all my own right art work depended not on my love of art, but of mountains and sea. . . . And through the whole of following life, whatever power of judgment I have obtained in art has depended on my steady habit of always looking for the subject principally, and for the art only as the means of expressing it." — John Ruskin.

and growing power. Growth in intelligence is by insensible degrees indicated by no exact demarkations.

Because a child cannot entertain the botanist's idea of a flower, shall we close the garden to him and confine his attention to one or even to half a dozen specimens? If not, why pursue such a course in mathematics? It is quite true that to "bring a bit of this thing and a bit of that thing transiently before the mind, creating ideas like dissolving views, each in its turn displacing its predecessor," is mischievous. It tends to intellectual chaos. Such a course is widely removed from leading pupils to discover common traits, to grow into the power of thinking of things not present to sense by bringing relations again and again before the mind in a variety of forms. The Many should reveal the One, as the mind passes from lower to higher phases of activity.

Restriction to a few things hampers the mind by forms, destroys elasticity, interferes with free mental imagery, with the progressive activity which, in due time, leads to seeing things as they are. Outside of school the child advances through an infinity of experiences. In school he advances in the same way if he advances at all. The environment should be so fresh, so varied, that the successive interests and growing powers of the pupil are fully provided for. Out of school the variety of nature does not confuse because it is not *pressed* upon the attention; it awaits our seeking. In no subject should we expect the child to move from fact to fact according to the logic of the adult. Let his approach to truth be free and natural. "The wholesome

idea does not attain at once its full expansion." A mistaken zeal defeats itself by efforts to urge [1] and help [2]

[1] "In dealing with the material world around him, the child can observe, search, and find what is not immediately apparent to look or touch; and he can discover relations between the different parts of a thing, or of one thing to other things. All this may be his own work, may be produced by self-activity. But the interest a child can take in observing is limited and of short duration." — Courthope Bowen.

[2] "One of the commonest errors, as well as one of the most pernicious, is that of assisting the child too much in all his work. . . . The kindergartener often forms his sentences for the child, overdirects him when he is matching colors, moves his blocks, sticks, tablets into more accurate position, and all for what reason? Primarily, to produce a better effect, it is probable, glorying in the consciousness that the work on every child's table is exactly right, and blind to the truth that uniformity must always be mechanical. . . . This vice, for it is a vice, of assisting the child too much causes him to lose his own power and makes him weak and dependent." — Kate Douglas Wiggin.

"We often find that when teachers fancy their pupils have obtained a thorough mastery of a subject, they are deceived, because they have not noticed that, in almost imperceptible ways, they have been doing for the pupil what he ought to be doing for himself. I have repeatedly gone into a school, and on examining it, say in arithmetic, have been told by the master, 'It is very strange that the boys do not know it; I thought they knew it thoroughly.' I have always asked them this: 'When you have examined them, have you made them answer for themselves?' And the reply has been, 'Yes, I have left them with themselves except just the very slightest possible help occasionally; just enough to prevent them from wandering about.' That is the whole thing. That very little help is the thing which vitiated the examination altogether; and the test of real mastery is that the knowledge shall be produced (and therefore obtained) without any help at all. When men and women in after-life come to use their knowledge, they will find that the knowledge is really of no use unless they are able to apply it absolutely without assistance, and without the slightest guidance to prevent them falling into the most grievous mistakes." — Bishop of Exeter.

the child. Interest dies when the self-active powers of sense and reason are not called forth.

Time.[1] — The time in which a given amount of work should be done varies with the class and the individual. Work above the ability weakens; so does work below it. A fixed amount in a fixed time is fatal.

Primary Object. — If this introduction seems unsatisfactory to those whose inquiries have led to its being written, it may not be amiss to say that many difficulties disappear when we keep steadily in view the development of the child. Meet the demands of education and we shall meet the demands of mathematics.

Economic work implies the quickening of the faculties[2] in all lines of effort. Reading, music, physical culture, nature study, mathematics all should help to train the eye to see quickly and correctly. Varied activities provide against overstrain, give greater freedom and greater self-control. Doing follows so naturally upon seeing in mathematics that drawing and making should need no urging. The visibility of the mind is to be insisted on in expression by hand as well as in expression through language. There is a natural order in muscle development as well as in mental development. To tax any sense to the neglect of others, or to exercise it along unnatural lines, is harmful. Neither the eye nor the hand nor the mind of the young child is ready for fine, exact work.

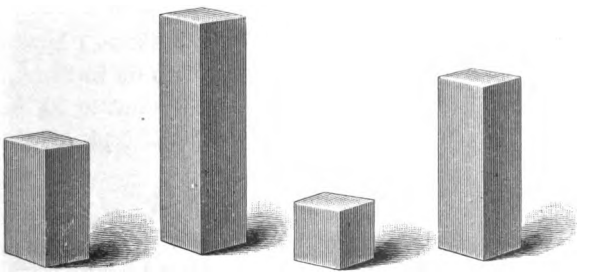
[1] See Part II, page 135.

[2] "The senses, and each one in particular, can be submitted to training by which their primordial capability may be indefinitely intellectualized." — Periere.

The aid of sensory and motor activity is to be constantly sought. Far from lessening the bodily power, the school should help to bring all activity into right relations. The dawning powers are fitly nourished through happy experiences which deepen the interest and strengthen the will through right conduct.

If hygiene and happiness seem foreign to the teaching of mathematics, the connection should be established. We shall not be successful in teaching any subject if we attempt to separate the thinking being from the being who senses, feels, and acts.

A list of problems is presented below in the hope that their study may bring more clearly into view the unity of the subject. The presentation is from the standpoint of the adult mind. To pursue such a plan with children would be destructive of growing power to analyze or relate. Such power cannot be forced.[1]



1. What is the relation of *b* to each of the other units? What is the relation of *b* to *a*?

2 is the relation of what to *a*?

[1] See remarks on evil effects of over-questioning, pages v, vi, Teachers' Book.

4 is the relation of what to c?
$\frac{1}{2}$ is the relation of what to a?
$\frac{3}{2}$ is the relation of what to a?
Call c 4. What is the name of each of the other units?
What is the relation of 16 to 8?
2 is the relation of what to 8?
4 is the relation of what to 4?
$\frac{1}{2}$ is the relation of what to 8?
$\frac{3}{2}$ is the relation of what to 8?
$\frac{3}{4}$ is the relation of what to 16?

2. 4 oranges cost 9¢. What is the cost of 1 doz. oranges?

$3 \cdot 9¢ = ?$

What is the relation of 1 doz. oranges to 4 oranges?
3 is the relation of what to 9¢?
What, then, is the cost of 1 doz. oranges?

3. A bushel of wheat weighs 60 lb. What is the weight of 3 pk. of wheat?

$\frac{15}{3 \cdot \cancel{60}} = ?$
$\cancel{4}$

What is the relation of 3 pk. to 1 bu.?
$\frac{1}{4}$ is the relation of what to 60 lb.?
$\frac{3}{4}$, then, is the relation of what to 60 lb.?
What, then, is the weight of 3 pk. of wheat?

4. What is the relation of 84 to 16?

$\frac{21}{\cancel{84}} = 5\frac{1}{4}.$
$\cancel{16}$
4

What is $\frac{1}{4}$ of 84? of 16?
What is the relation of 21 to 4?
What, then, of 84 to 16?

5. $\frac{5}{4}$ is the relation of what to 24?

$\frac{6}{5 \cdot \cancel{24}} = 30.$
$\cancel{4}$

$\frac{1}{4}$ is the relation of what to 24?
$\frac{5}{4}$, then, is the relation of what to 24?

6. 4 acres of land cost \$840. What is the cost of 6 acres?

$$\frac{3 \cdot \overset{420}{\cancel{\$840}}}{\cancel{2}} = 1260.$$

What is the relation of 6 to 4?
$\frac{1}{2}$ is the relation of what to 840?
$\frac{3}{2}$, then, is the relation of what to 840?
What, then, is the cost of 6 acres.

7. What is the relation of $\frac{1}{8}$ to $\frac{3}{4}$?

Ans.: $\frac{1}{6}$ is the relation of $\frac{1}{8}$ to $\frac{3}{4}$.

8. What is the relation of 7 bu. to 5 pk.?

$$\frac{7 \cdot 4}{5} = \frac{28}{5} = 5\frac{3}{5}.$$

What is the relation of 1 bu. to 5 pk.?
What, then, *equals* the relation of 7 bu. to 5 pk.?
What, then, *is* the relation of 7 bu. to 5 pk.?

9. What is the relation of $\frac{2}{3}$ to $\frac{3}{4}$?

$$\frac{2 \cdot 4}{3 \cdot 3} = \frac{8}{9}.$$

What is the relation of 1 to $\frac{3}{4}$?
What, then, *equals* the relation of $\frac{2}{3}$ to $\frac{3}{4}$?
What, then, is the relation of $\frac{2}{3}$ to $\frac{3}{4}$?

10. $\frac{2}{3}$ bbl. of flour costs \$4$\frac{2}{3}$. What is the cost of $\frac{3}{8}$ bbl.?

$$\frac{3 \cdot 3 \cdot 14}{8 \cdot 2 \cdot 3} = 2\frac{5}{8}.$$

What *equals* the cost of a barrel?
What, then, *equals* the cost of $\frac{3}{8}$ bbl.?
What *is* the cost of $\frac{3}{8}$ bbl.?

11. $11\frac{1}{3}$ tons of hay cost \$72. What is the cost of $12\frac{3}{4}$ tons?

$$\frac{\overset{3}{\cancel{51}} \cdot 3 \cdot \overset{9}{\cancel{72}}}{\underset{2}{\cancel{4}} \cdot \cancel{34}} = 81.$$

What is the relation of 1 ton to $11\frac{1}{3}$ tons?
What, then, equals the cost of 1 ton?
What, then, *equals* the cost of $12\frac{3}{4}$ tons?
What *is* the cost of $12\frac{3}{4}$ tons?

12. What is the relation of .98 to .7?

$$\frac{\cancel{98}}{\cancel{70}} = \frac{7}{5} = 1.4.$$

.7 equals how many hundredths?
What is the relation of .98 to .70?
What, then, is the relation of .98 to .7?

13. .7 of a bar of iron weighs 326 lb. What is the weight of .98 of the bar?

$$\frac{\overset{7}{\cancel{98}}\cdot 326}{\underset{5}{\cancel{70}}} = ?$$

.7 equals how many hundredths?
What is the relation of .98 to .70?
What, then, *equals* the weight of .98 of the bar?
What, then, *is* the weight of .98 of the bar?

14. .35 of a ton of coal costs $1.40. What is the cost of 6.5 tons?

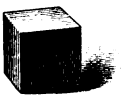
$$\frac{\overset{130}{\cancel{650}}\cdot \overset{.20}{\cancel{\$1.40}}}{\underset{7}{\cancel{\$5}}} = \$26.$$

6.5 equals how many hundredths?
What is the relation of 650 to 35?
What, then, is the relation of 6.5 to .35?
What, then, equals the cost of 6.5?
What, then, *is* the cost of 6.5?

15. How many acres in a park equal to .3 of 640 acres?

$$\frac{3\cdot 64\cancel{0}}{\cancel{10}} = ?$$

.1 is the relation of what to 640 acres?
.3, then, is the relation of what to 640 acres?
What, then, is the number of acres in the park?



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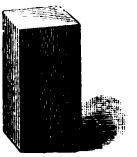


b

16. If a is 1, what is b?
What is the relation of $\frac{1}{3}$ of 2 to $\frac{1}{3}$ of 1?
Then $\frac{1}{3}$ of 2 equals how many thirds of 1?
Then $\frac{2}{3}$ of 1 equals $\frac{1}{3}$ of what?
What is the relation of $\frac{1}{4}$ of 3 to $\frac{1}{4}$ of 1?
Then $\frac{1}{4}$ of 3 equals how many fourths of 1?
Then $\frac{3}{4}$ of 1 equals $\frac{1}{4}$ of what?

$\frac{3}{5}$ of 12 equals $\frac{1}{5}$ of what?
$\frac{1}{5}$ of 3 12's equals $\frac{3}{5}$ of what?
What is the ratio of $\frac{3}{5}$ of 12 to $\frac{1}{5}$ of 3 · 12's?
$\frac{2}{3}$ of 7 equals $\frac{1}{3}$ of how many 7's?
$\frac{3}{4}$ of 100 apples equals $\frac{1}{4}$ of how many hundred apples?
$\frac{3}{4}$ of 100% equals $\frac{1}{4}$ of how many hundred per cent?
$\frac{2}{3}$ of x equals $\frac{1}{3}$ of what?
What part of x equals $\frac{1}{3}$ of $2x$?

17. $\frac{3}{5}$ of 1 equals what decimal of 1?

$\frac{3}{5}$ of 1 equals $\frac{1}{5}$ of what?
3 equals how many tenths of 1?
$\frac{1}{5}$ of 30 tenths equals what?
Then $\frac{3}{5}$ of 1, or $\frac{3}{5}$, equals what decimal of 1?

$$5\overline{)3.0} \quad .6$$

18. 3 equals what per cent of 4?

4 equals what per cent of 4?
Then 3 equals what per cent?

$$\frac{3 \cdot \overset{25\%}{\cancel{100\%}}}{\cancel{4}} = 75\%.$$

19. $1\frac{1}{2}$ equals what per cent of $2\frac{1}{2}$?

$2\frac{1}{2}$ equals what per cent of $2\frac{1}{2}$?
Then $1\frac{1}{2}$ equals what per cent?

$$\frac{3 \cdot \overset{20\%}{\cancel{100\%}}}{\cancel{5}} = 60\%.$$

20. $\frac{3}{7}$ equals what per cent of $\frac{3}{4}$?

$\frac{3}{4}$ equals what per cent of $\frac{3}{4}$?
Then 1 equals what per cent of $\frac{3}{4}$?
Then $\frac{3}{7}$ equals what per cent?

$$\frac{\cancel{3} \cdot 4 \cdot \overset{14\frac{2}{7}\%}{\cancel{100\%}}}{\cancel{7} \quad \cancel{3}} = 57\frac{1}{7}\%.$$

21. .03 equals what per cent of 1.7?

1.7 equals what per cent of 1.7?
Then .03 equals what per cent?
$\frac{3}{170}$ of 100% equals $\frac{1}{170}$ of what?

$$\frac{3 \cdot 100\%}{170} = \frac{30\%}{17} = 1\frac{13}{17}\%.$$

What is the ratio of .03 to 1.7? of 1.7 to .03?
$1\frac{13}{17}\%$ of 1.7 equals what?

22. 72 equals what per cent of 93?

$$93\overline{)7200\%}\quad 77\tfrac{13}{31}\%$$

$$\begin{array}{r} 651 \\ \hline 690 \\ 651 \\ \hline 39 \end{array}$$

93 equals what per cent of 93?
72 equals what per cent?
$\frac{13}{93}$ of 100% equals $\frac{1}{93}$ of what?

23. What equals $1\frac{3}{4}\%$ of 72 ft.?

$$\frac{7 \cdot 18}{4} = 1.26.$$

What is 1% of 72 ft.?
What, then, equals $\frac{1}{4}\%$?
What, then, equals $\frac{7}{4}\%$, or $1\frac{3}{4}\%$?

24. 85 bu. equal $\frac{3}{4}\%$ of what?

$$\frac{100 \cdot 4 \cdot 85}{3} = ?$$

$\frac{3}{4}\%$ equals what?
Then 1% equals what?
Then what does 100% equal?
$\frac{3}{4}\%$ is the relation of 85 bu. to what?

25. A schoolhouse was insured at $\frac{4}{5}\%$. The premium was \$18.75. For how much was the house insured?

$$\frac{100 \cdot 5 \cdot 18.75}{4} = ?$$

$\frac{4}{5}\%$ equals what?
Then 1% equals what?
Then 100% equals what?
What, then, equals the amount for which the house was insured?

26. A man sold a horse for \$500, which was a gain of 25%. Find cost.

$$\frac{4 \cdot 500}{5} = ?$$

What is the relation of cost to selling price?

27. A man paid \$300 for a horse and sold it at a gain of 20%. Find selling price.

$$\frac{6 \cdot 300}{5} = ?$$

What is the relation of selling price to cost?

28. I paid all of my money for a buggy. Had I paid $\frac{8}{8}$ more the buggy would have cost \$209. Find cost.

$\frac{8 \cdot 209}{11} = ?$ What is the relation of real cost to supposed cost?

29. What is the number of square inches in a rectangle whose base is 17″ and whose altitude is $2\frac{1}{2}$″?

$\frac{5 \cdot 17}{2}$ = the number. What is the number in the rectangle if the altitude is 1″? if $\frac{1}{2}$″? if $\frac{5}{2}$″, or $2\frac{1}{2}$″?

30. What is the number of cubic inches in a solid whose base is 9″ by 13″ and whose altitude is $4\frac{2}{3}$″?

$\frac{14 \cdot 9 \cdot 13}{3}$ = the number. What equals the number in the solid if 1″ is the altitude? if $\frac{1}{3}$″? if $\frac{14}{3}$″, or $4\frac{2}{3}$″?

31. What is the number of square feet in a rectangle whose base is $5\frac{3}{4}$ ft. and whose altitude is $19\frac{1}{3}$ ft.?

$\frac{58 \cdot 23}{3 \cdot 4} = ?$ What is the number in the rectangle if the altitude is 1 ft.? if $\frac{1}{3}$ ft.? if $\frac{58}{3}$ ft.?

32. If 172 is the number of square inches in a rectangle 18″ long, what is its altitude?

$\frac{172}{18}$ = the number of units in the altitude.

If *18* were the *number* of square inches in a rectangle 18 in. long, what would be its altitude?

If *1* were the *number*, what would be its altitude?

What, then, equals the altitude if 172 is the number?

33. 784 is the number of cubic feet in a solid 2′ by 3′. What is its altitude?

$\frac{784}{2 \cdot 3} = ?$ If *1* were the number of cubic feet in the solid, what would be its altitude?

What, then, equals the altitude if 784 is the number?

34. How many cords of wood in a pile 6 ft. high, 4 ft. wide, and 70 ft. long?

$$\frac{6 \cdot 4 \cdot 70}{8 \cdot 16} = \text{the number of cords.}$$

What equals the number of cubic feet in the pile? the number of cord feet? the number of cords?

35. How many cubic yards in a cellar $8\frac{1}{2}$ ft. deep, 22 ft. wide, and 36 ft. long?

$$\frac{17 \cdot 22 \cdot 36}{2 \cdot 27} = \text{the number of cubic yards.}$$

$\frac{1}{2}$ of $22 \cdot 36$ equals the number of what?

The *number* of cubic yards equals what part of the number of cubic feet?

36. How many barrels of flour at \$6 a barrel must be given for 24 tons of coal at \$5 a ton?

$$\frac{4}{\cancel{24} \cdot 5}{\cancel{6}} = 20.$$

Price ton of coal equals what part price barrel flour?

Then the price of 24 tons of coal equals what?

37. A post 7 ft. high casts a shadow 3 ft. long. What is the height of a telegraph pole that casts an 18-ft. shadow?

$$\frac{6}{\cancel{18} \cdot 7}{\cancel{3}} = 42.$$

What is the relation of the shadow of the telegraph pole to the shadow of the post?

6 is the ratio of what to 7 ft., the height of the post?

38. If a ship has sufficient water to last a crew of 25 men 8 mo., how long will it last 15 men?

$$\frac{5}{\cancel{25} \cdot 8}{\cancel{15}}{3} = ?$$

15 men will use what part of the water in 8 mo.?

What is the ratio of the water to $\frac{3}{5}$ of it?

What, then, equals the time it will last 15 men?

What, then, is the time?

39. If \$250 yields \$17.50 interest in 1 yr., what, at the same rate, will \$450 yield in 8 mo.?

$$\frac{2 \cdot 9 \cdot 17.50}{3 \cdot 5} = ?$$

What is the relation of \$450 to \$250?
What, then, will \$450 yield in 1 yr.?
What, then, will it yield in 8 mo.?

40. If 15 men in 18 dy. of 10 hr. each cut 60 cords of wood, how many cords can 40 men cut in 24 dy. of 9 hr. each?

$$\frac{40 \cdot 24 \cdot 9 \cdot 60}{15 \cdot 18 \cdot 10} = ?$$

15 men in 18 dy. of 10 hr. each can cut how much wood?
How much, then, can 15 men cut in 24 dy. of 9 hr. each?
What, then, can 40 men cut in the same time?

41. If a 5¢ loaf of bread weighs 12 oz. when flour is \$8 a barrel, how much should a 3¢ loaf weigh when flour is \$9 a barrel?

$$\frac{3 \cdot 8 \ 12}{5 \cdot 9} = ?$$

When flour is \$9 a barrel, how much can be bought for \$8?
What, then, should a 5¢ loaf weigh when flour is \$9?
What, then, should a 3¢ loaf weigh?

42. How many yards of cloth 27 in. wide are necessary to line x yd. a yard wide?

$$\frac{4 \cdot x}{3} = \text{number of yards.}$$

43. If a is the number of feet in the length of a room, b in the width, and c in the height, what is the number of square yards in the walls of the room?

$$\frac{2c(a+b)}{9} = \text{number of square yards.}$$

44. If a is the diameter of a circle, what is its area ?

$$\frac{11 \cdot a^2}{14} = \text{area.}$$

45. If b is the number of squares in the surface of a cube, what is its length ?

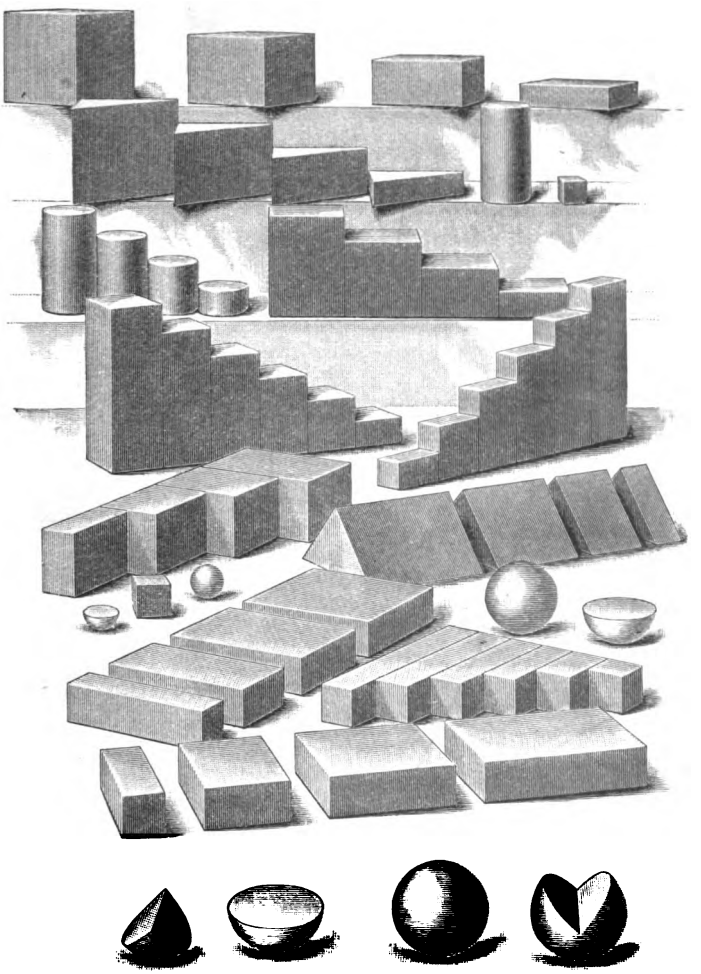
$$\sqrt{\frac{b}{6}} = \text{length of cube.}$$

46. What is the interest of \$84, at 8%, for 1 yr. 3 mo. ?

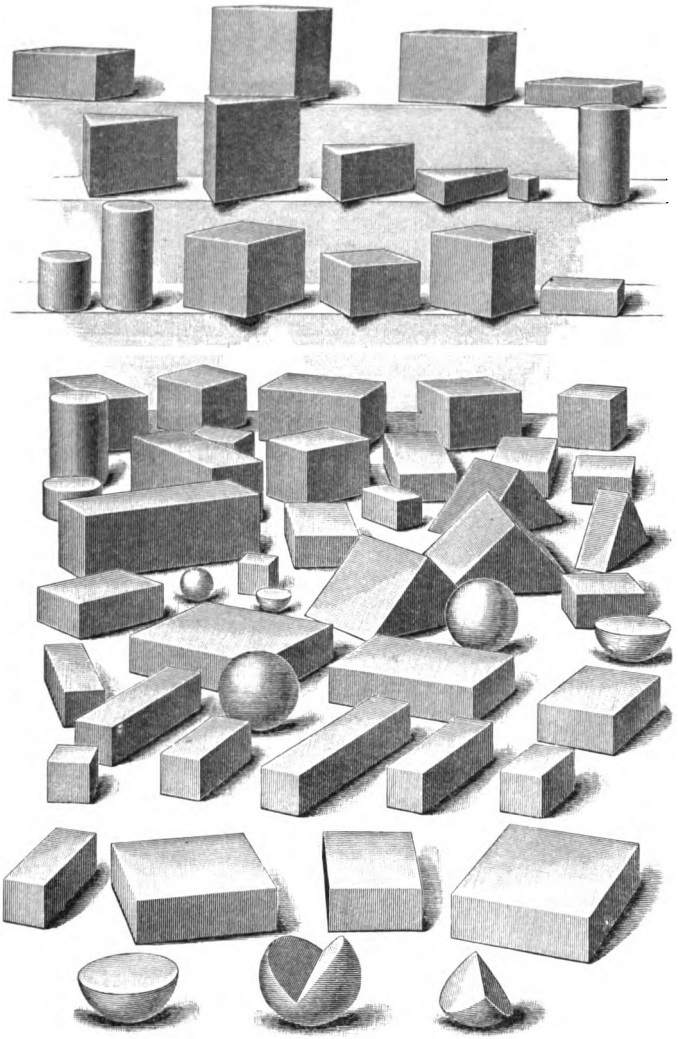
$$\frac{4 \cdot 15 \cdot \$.84}{3 \cdot 2} = \text{int. of \$84 for 1 yr. 3 mo. at 8\%.}$$

47. What is the interest of \$875, at 7½%, for 9 mo. 12 dy. ?

$$\frac{5 \cdot 9.4 \cdot \$8.75}{4 \cdot 2} = \text{int. of \$875 at 7½\% for 9 mo. 12 dy.}$$



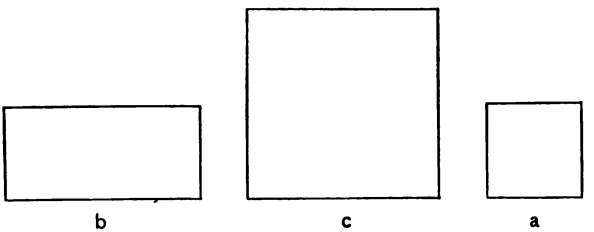
Blocks illustrating integral and fractional relations.



Blocks illustrating integral and fractional relations.

AN ELEMENTARY ARITHMETIC.

PART I.



Ratios. — 1. Tell what you can about a, b, and c.

2. b equals how many times a? c equals how many times b? c equals how many times a?

3. a equals what part of b? What part of c equals a?

4. If a is 1, what is b? what is c?

5. If c is 1, b equals what part of 1?

6. If c is worth 40¢, what part of 40¢ is each of the others worth?

If a is worth 10¢, how many 10¢ is each of the others worth?

7. If the length of a is 1, what is the length of b? of c? What is the width of b?

8. Show me the square whose length is 1; the square whose length is 2. The square of 1 equals what part of the square of 2?

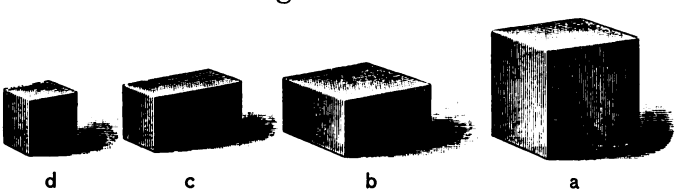
9. Draw a line and call it 1; draw the line 2. Draw the square of the 1; of the 2. The square of the 1 equals what part of the square of the 2?

10. Draw a line 6 in. long on the blackboard and call it 1; draw 2. Draw the square of the 1 and of the 2. Compare the squares.

11. Cut a square of 1 and a square of 2. Compare.

12. Draw the rectangles *a*, *b*, *c* on the blackboard, making *a* 6 in. long. Review.

13. Draw rectangles again, making *a* 1 ft. long and *b* and *c* 2 ft. long. Review.



Place the cube 1 in. long, the cube 2 in. long, the solid 2 in. by 2 in. by 1 in., and the solid 2 in. by 1 in. by 1 in. where they can be handled. Call the solids *d*, *c*, *b*, and *a*.

1. Tell all you can about the relations of these solids.

2. The cube *a* can be cut into how many solids equal to *b*?

3. *a* equals how many times *b*? What part of *b* is as large as *c*? *b* equals how many times *c*? What part of *c* is as large as *d*? *c* equals how many times *d*?

4. If b weighs 4 oz., what does each of the other solids weigh?

5. If c holds 6 pt., what part of 6 pt. does d hold? How many 6 pt. does each of the others hold?

6. Compare d with c. Compare d with b.

7. Compare c with each of the others.

Example: $c = 2$ times d; $\frac{b}{2}$ (read, $\frac{1}{2}$ of b); $\frac{a}{4}$.

8. Compare b with each of the others.

9. Compare a with b. Compare a with c.

10. If the length of d is 1, what is the length of a?

11. If the length of a is 1, what is the length of b?

(See cut on opposite page.)

1. Which unit is 2 times as large as b?

2. Which unit is 4 times as large as d?

3. Which unit equals $\frac{1}{2}$ of a?

4. Which equals $\frac{1}{4}$ of b?

5. 2 is the relation of which unit to b?

Answer: 2 is the relation of a to b.

6. 2 is the relation of which unit to c?

7. 2 is the relation of which unit to d?

8. 4 is the relation of which unit to d?

9. $\frac{1}{2}$ is the relation of which unit to c? of which unit to a? of which unit to b?

10. $\frac{1}{4}$ is the relation of which unit to a?

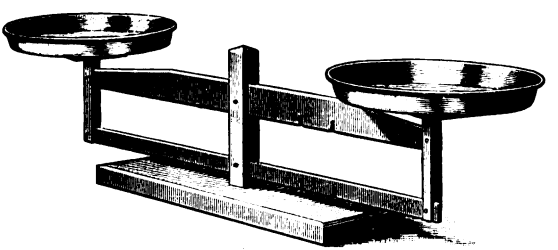
11. Place the 2-in. cube, the 4-in. cube, the solid 4 in. by 4 in. by 2 in., and the solid 4 in. by 2 in.

by 2 in. where they can be handled. Compare these solids in like manner.

Weight and Muscular Sense. — 1. Handle two objects, and tell which is the heavier or which is the lighter.

Ex.: The weight of this sphere is not equal to the weight of that piece of sandstone.

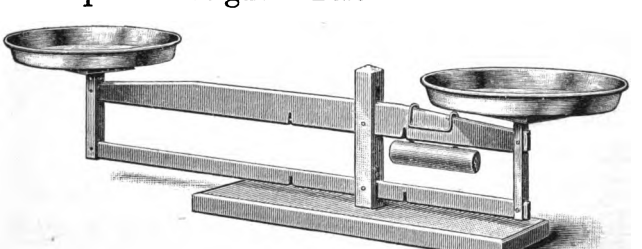
Test the estimates by putting the objects into the scale-pans.



Give many exercises and secure variety of expression.

2. Select objects which you think equal in weight, or the ratio of whose weight is 1. Test.

3. Select objects of different sizes which you think equal in weight. Test.



Give many exercises.

4. Handle objects, and judge whether the ratio of the weight is more or less than 2. Test.

5. Handle objects, and judge whether the ratio of the weight is more or less than $\frac{1}{2}$.

Ex.: The weight of this piece of soft wood is less than $\frac{1}{2}$ of the weight of that piece of hard wood.

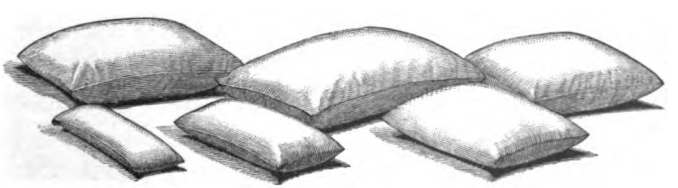
6. Find objects whose weight ratio is 2. Test.

7. Find objects whose weight ratio is $\frac{1}{2}$. Test.

8. Try to find objects equal in size whose weight ratio is about 2. Test.

9. Try to find equal objects whose weight ratio is about $\frac{1}{2}$. Test.

Secure variety of expression. Give many exercises.



Make bags[1] holding 8 oz., 1 lb., 2 lb., 3 lb., 4 lb. and 5 lb. of coarse sand or of other material. Pupils handle different weights. Teach the name of each.

1. Unite different weights and tell the sum. Tell differences.

2. Make comparison of the different weights.

[1] A pretty covering for the bags will add interest to the exercises.

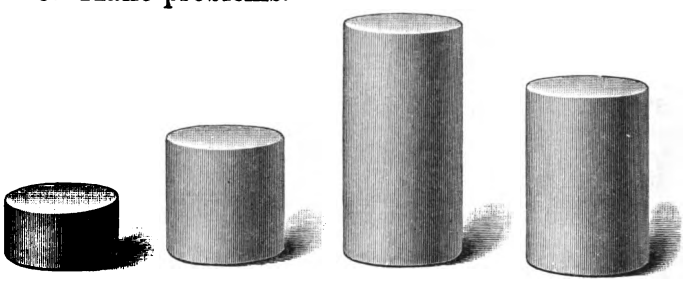
3. Pupils close or cover eyes. Place different weights in hand. Pupils estimate weight. Tell correct weight after each estimate.

4. Handle solids, and tell whether they weigh more or less than certain weights.

Ex.: This block weighs less than 1 lb. This limestone weighs about 3 lb.

5. Repeat and vary these exercises.

6. Make problems.



1. If the largest unit represents 1 lb., what part of 1 lb. is represented by each of the other units?

2. Compare $\frac{1}{4}$ lb. with each of the others; compare $\frac{1}{2}$ lb.; compare $\frac{3}{4}$ lb.; compare 1 lb.

3. There are 4 oz. in $\frac{1}{4}$ lb. How many oz. in $\frac{1}{2}$ lb.? in $\frac{3}{4}$ lb.? in 1 lb.?

4. Compare 4 oz. with each of the others. Compare 8 oz. with each of the others. Compare 12 oz. with each of the others. Compare 1 lb. with 4 oz.; 8 oz.; 12 oz.

5. $\frac{1}{2}$ is the relation of what to 1 lb.?

Ans.: $\frac{1}{2}$ is the relation of 8 oz. to 1 lb.

6. $\frac{1}{2}$ is the relation of what to 16 oz.?
7. $\frac{1}{2}$ is the relation of what to 8 oz.?
8. $\frac{2}{3}$ is the relation of what to 12 oz.?
9. Which unit is 3 times as large as $\frac{1}{2}$ of 8 oz.?
10. Observe units, and ask and answer questions.
11. Review without observing units.
12. If you have 1 lb. of salt, how many cups each holding $\frac{1}{4}$ lb. can you fill? how many 8-oz. cups?
13. If you call the smallest block $\frac{1}{4}$ lb., what should you call each of the others? if you call the smallest block 1 lb.? if you call it 4 oz.?
14. 3 lb. equal how many 4ths of 4 lb.?
15. 12 oz. equal how many 4ths of 16 oz.?
16. The cake that can be made with 3 lb. of sugar equals what part of the cake that can be made with 4 lb.?
17. If you eat $\frac{1}{2}$ lb. of butter in a week, $\frac{3}{4}$ lb. will last you how long?

Problems. — 1. At 5¢ a pound, what is the cost of 3 lb. of sugar?

2. 8¢ is the cost of 1 lb. of sugar; what is the cost of $\frac{1}{2}$ lb.? of $\frac{1}{4}$ lb.? of $\frac{3}{4}$ lb.?

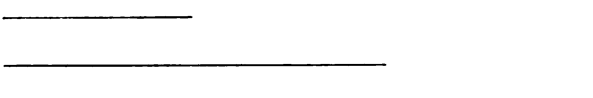
3. If $\frac{1}{2}$ lb. of coffee costs 20¢, what is the cost of $\frac{1}{4}$ lb.? of 1 lb.?

4. A package of tea weighs 1 lb., another package 4 oz. How much more does the first package weigh than the second?

5. The larger package can be made into how many packages the size of the smaller?

6. If 4 oz. of tea cost 15¢, what is the cost of $\frac{1}{2}$ lb.?

Yard and Foot. — 1. Practice drawing groups of lines, 1 yd., $\frac{2}{3}$ yd., and $\frac{1}{3}$ yd. long. Measure



the lines in each group. How many feet in each line? How many inches in 1 ft? in 2 ft.? in 1 yd.?

2. Compare 12 in. with 2 ft.; with 1 yd.

3. Compare 24 in. with 12 in.; with 1 yd.

4. Compare 1 yd. with 12 in.; with 24 in.

5. How many 6 in. in 1 ft.? in 2 ft.? in 1 yd.?

6. How many 6 in. in $\frac{1}{3}$ yd.? in $\frac{1}{2}$ yd.? in $\frac{2}{3}$ yd.?

7. Compare 6 in. with $\frac{1}{3}$ yd.; with $\frac{1}{2}$ yd.; with $\frac{2}{3}$ yd.; with 1 yd.

8. Compare 1 ft. with 6 in.; with $\frac{1}{2}$ yd.; with $\frac{2}{3}$ yd.; with 1 yd.

9. Cover the lines and review.

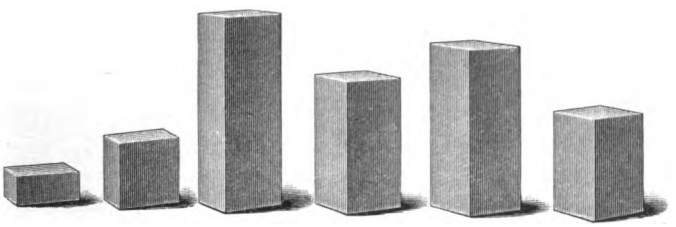
Problems. — 1. A yard of wire at 2¢ a foot costs how much?

2. When you buy $\frac{2}{3}$ of a yard of cloth at \$1 a yard, what part of \$1 do you pay for the cloth?

3. A square rug is 3 ft. long. How many yards of braid will be required to bind it?

4. How many 6 in. in 1 ft.? How many 6 in. in $\frac{2}{3}$ of a yard? Draw a line equal to $\frac{2}{3}$ of a yard.

5. A yard of twine will reach how many times as far as 1 ft.? 2 yds. of twine will reach how many times as far as 1 ft.?



Ratios. — 1. Select solids having the relation, or ratio, of *a*, *b*, *c*, *d*, *o*, *e*.

2. Name the solids *a*, *b*, *c*, *d*, *o*, *e*.

The means of expressing must be as freely supplied as the means of discovery. The pupil is not expected to invent terms.

3. Tell all you can about the relation of these units.

4. Unite units and tell what the sum equals.

5. Make statements like this: *o* less *e* equals *b*.

6. *c* can be separated into how many *d*'s? into how many *b*'s?

7. c can be separated into how many b's? What is the name of the largest unit that can be found in both c and d an exact number of times?

8. Each of the other units equals what part of c?

9. If b is 1, what is each of the other units?

10. If a is 1, what is each of the other units?

11. If b is 1, how many 1's are there in each of the other units?

12. If d is 1, how many 1's and parts of 1 in each of the other units?

13. 2 is the relation of what units?

14. 3 is the relation of what units?

15. $\frac{1}{2}$ is the relation of what units?

16. $\frac{2}{3}$ is the relation of what units?

17. Which units have the relation $\frac{3}{2}$?

18. Which unit is 3 times as large as $\frac{1}{2}$ of b?

19. c equals 6 times $\frac{1}{3}$ of what unit?

20. $\frac{1}{3}$ of what unit equals $\frac{1}{6}$ of c?

21. What equals $\frac{1}{2}$ of c? d equals how many sixths of c?

22. o equals 5 times $\frac{1}{3}$ of what unit?

23. $\frac{1}{3}$ of what unit equals $\frac{1}{5}$ of o?

24. $\frac{2}{3}$ of d equals what unit? b equals how many thirds of d?

25. 2 is the ratio of d to $\frac{1}{3}$ of what unit? 3 is the ratio of d to $\frac{1}{2}$ of what unit?

26. d equals $\frac{3}{4}$ of what unit? $\frac{3}{4}$ is the ratio of what units?

1. If a ribbon 12 in. long reaches from the top to the bottom of *c*, what part of this ribbon reaches from top to bottom of *d*? of *e*? of *o*?

2. If 8 caramels fill a box as large as *e*, how many are needed to fill boxes as large as each of the others?

3. If *a* is worth 2¢, how many 2¢ is each of the others worth?

4. If we make 2 of *a*, how many 2's can we make of each of the other units?

If we call *a* 2, what should we call each of the other units?

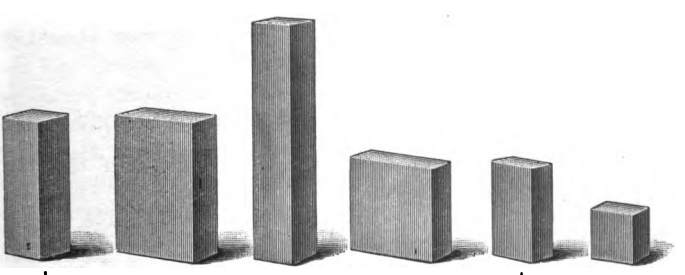
Show me the *2*, the *12*, the *8*, etc.

5. 2 equals what part of each of the other units?

6. What is the relation of *4* to each of the other units? of *6*? of *8*? of *10*? of *12*?

7. If *a* represents 3 lb. of sugar, how many 3 lb. do each of the other units represent?

8. If *a* is worth 13¢, each of the other units is worth how many 13¢?



Use another set of solids having the same ratios. Call the solids *d*, *c*, *o*, *e*, *b*, *a*.

Ratios. — 1. Tell all you can about the relations of these units.

Call the solids *2*, *4*, *6*, *8*, *10*, *12*.

2. Find me the *6*, the *12*, the *4*, etc.

3. Give the relations of each unit to every other unit of the set.

Review without solids.[1]

4. What is the ratio of 8 to 4? 2 is the ratio what to 4?

5. What is the ratio of 4 to 6? of 6 to 4?

6. $\frac{2}{3}$ is the ratio of what to 6? $\frac{3}{2}$ is the ratio of what to 4? $\frac{3}{2}$ is the ratio of 6 to what?

Problems. — 1. If 4 oranges cost $x¢$, how many halves of $x¢$ will 6 oranges cost?

2. If 4 oranges cost $20¢$, what do 6 oranges cost? 1 doz. oranges?

3. If 8 pencils cost $12¢$, what do 4 cost?

4. What is the ratio of 10 to 6?

[1] Exercises similar to those with objects in view should be carried on immediately after the removal of the objects, and also after intervals of time. Gradually impressions should persist, so that the thought will require less outward stimulus. This is true not only of the thought expressed by language, but of that which is expressed by drawing, cutting, making, etc. Attention is the indispensable condition of forming and fixing impressions. Children *can* attend for a short time only.

"The first foundation of this (constructiveness) may be called memory, provided we understand that it is memory of the concrete, or the full sensible image of the things that have impressed the senses." — Professor Bain.

5. If a boy earns $6 a week, in what time can he earn $10?

6. If cherries sell for 10¢ a box, and strawberries for $\frac{3}{2}$ as much, what is the cost of a box of strawberries?

7. If 12 lumps of sugar sweeten 1 qt. of coffee, what part of 1 qt. do 8 lumps sweeten?

8. 4 lumps and what are 12 lumps? 8 lumps and what are 12 lumps?

9. If a boy has 2¢, how many more does he need to have 10¢?

10. Edward has 4¢ and William 6¢. What sum have both?

11. James had 10¢ and paid $\frac{1}{5}$ of it for a pencil. How much did he pay for the pencil? What part of 10¢ must he pay for 3 pencils? Why?

12. If a man can build a shed in 10 days, what part of it can he build in 2 days?

13. How many feet in the perimeter of a rug 1 yd. long and 2 ft. wide?

14. If 10 oranges cost 50¢, what part of 50¢ is the cost of 8 oranges?

15. What is the ratio of 10 to 2?

A boy earns a dollar in 2 hours. How much does he earn in 10 hours?

16. Frank has 12¢. My money equals $\frac{2}{3}$ of his. How much have I?

17. If 12 boxes can be put into a basket, how many can be put into a basket $\frac{2}{3}$ as large?

18. Clarence can walk a mile in $\frac{1}{2}$ hour. How far can he walk in 2 hours?

19. I can buy 2 oranges for $\frac{1}{5}$ of my money. How many can I buy for my money?

20. What is the ratio of my money to $\frac{1}{5}$ of my money?

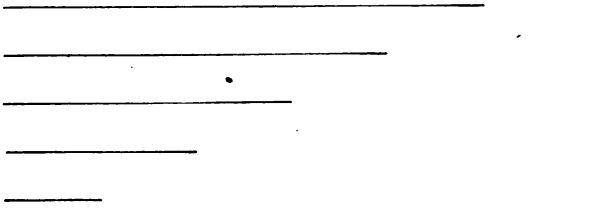
21. What is the ratio of the candy I can buy for my money to the candy I can buy for $\frac{1}{5}$ of my money?

22. Which will cost more: $\frac{1}{2}$ of 4 apples or $\frac{1}{5}$ of 10 apples?

23. If for $\frac{1}{2}$ of 4 apples I pay a nickel, how much must I pay for $\frac{1}{5}$ of 10 apples?

24. If for $\frac{1}{2}$ of 4 apples I pay $x\not{e}$, how much must I pay for $\frac{1}{5}$ of 10 apples? for $\frac{3}{5}$ of 10 apples?

Ratios of Time. — 1. Draw lines representing 2 mo., 4 mo., 6 mo., 8 mo., 10 mo., 1 yr.



2. Point to the different lines and tell what each represents.

3. Tell all you can about the relations of these periods of time.

4. Each line represents what part of a year?

5. Make sentences like this: The sum of 2 mo. and 4 mo. equals 6 mo.

6. What is the relation of 2 mo. to each of the other periods of time? of 4 mo.? of 6 mo.? of 8 mo.? of 10 mo.? of 1 yr.?

7. How many 6 mo. in a year? how many 4 mo.? how many 3 mo.? how many 2 mo.?

8. 6 mo. equals what part of a year? 4 mo. equals what part of a year? 2 mo. equals what part of a year?

9. What equals $\frac{1}{2}$ yr.? $\frac{1}{3}$ yr.? $\frac{1}{4}$ yr.? $\frac{1}{6}$ yr.? $\frac{2}{3}$ yr.? $\frac{3}{4}$ yr.?

10. Make sentences like this: 2 is the relation of 4 mo. to 2 mo.; of 8 mo. to 4 mo.

11. 3 is the relation of what to 2 mo.? of what to 4 mo.?

12. $\frac{2}{3}$ is the relation of what to 1 yr.? of what to $\frac{1}{2}$ yr.?

13. If a boy lives in a city 10 mo., what part of the year does he live in the country?

14. The rent of a house for 3 mo. is \$100. How many hundred dollars is the rent for 1 yr.?

15. Willie spent $\frac{2}{3}$ of a year with his aunt in Boston. How many months did he spend there?

16. Review again and again.

17. Cover the lines and review.

1. What is the ratio of 1 yr. to 2 mo.? to 2·2 mo.? to 3·2 mo.? to 4·2 mo.? to 5·2 mo.?

2. Fred puts an equal amount of money into his bank every month. At the end of the year he will have how many times as much as at the end of 6 mo.? as at the end of 3 mo.?

3. If he saves $\$\frac{1}{2}$ a month, how many $\$\frac{1}{2}$ will he save in 1 yr.? how many dollars?

4. If Fanny reads a book every month, how many does she read in $\frac{2}{3}$ of a year?

5. $\frac{2}{3}$ of a year equals how many times 4 mo.?

The money a man earns in 8 mo. equals how many times the money he earns in $\frac{1}{3}$ of a year?

6. If you attend school 10 mo. of a year, how many months' vacation do you have?

7. How many more months of school than of vacation?

8. 2 mo. equals what part of 10 mo.? 6 mo equals what part of 10 mo.?

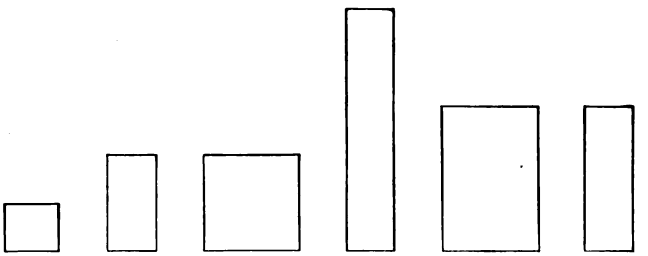
9. Frank lived in St. Louis 4 mo., which equals $\frac{1}{3}$ of the time he lived in Chicago. How long did he live in Chicago?

10. Lucy is 10 yr. old and Carrie 4 yr. Lucy is how many years older than Carrie? Carrie is how much younger than Lucy? In how many years will Carrie be as old as Lucy now is?

Ratios. — 1. Practice drawing squares $\frac{1}{2}$ ft. long and 1 ft. long.

Cut small squares having the same ratio.

2. Draw the rectangles a, b, c, etc., on the blackboard, making a $\frac{1}{2}$ ft. long.



3. Tell all you can about the relations of these rectangles.

4. Each rectangle can be cut into how many squares equal to a?

5. The sum of b and c equals what? of a and b? of f and a? of b and f? of a and d?

6. Make sentences like this: c is b less than e.

7. Compare a with each of the other units.

8. What part of b equals a? What part of each of the other units equals a?

9. b equals what part of f? What part of each of the other units equals f? What is the relation of each unit to f?

10. b equals what part of c? c can be cut into how many a's? What part of c equals f? f equals what part of c?

11. What part of d equals c? What part of e equals c? Compare c with each of the other units.

12. What is the relation of each unit to *e*? of *e* to each of the other units?

13. Cover the units. Tell all you can about the relations of the units.

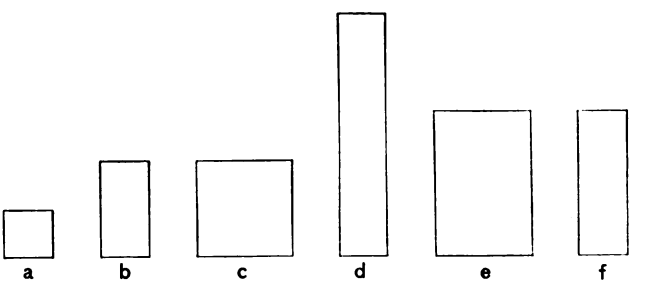
14. $\frac{1}{2}$ is the relation of which units?

15. 2 is the relation of which units?

16. $\frac{2}{3}$ is the relation of which units?

17. $\frac{3}{2}$ is the relation of which units?

18. Which unit is 5 times as large as $\frac{1}{2}$ of *b*?



1. If *a* is 2, how many 2's are there in each of the other units?

2. If we call *a* 2, what name should we give to each of the other units?

3. Show me the *4*, *6*, *8*, *10*, *12*. Point to each and tell its name again.

4. Compare *2* with each of the other units.

5. Compare *4* with each of the other units.

6. Compare *6*, *8*, *10*, *12* with each of the other units.

Continue the work until pupils become familiar with the relations of *2*, *4*, *6*, etc.

1. If e is 1, what part of 1 do you see in each of the other units?

2. If b is 1, what is each of the other units?

3. If f is $\frac{1}{2}$, each of the other units equals what?

Ex.: $a = \frac{1}{3}$ of the $\frac{1}{2}$.

4. If a is $\frac{1}{3}$, what is each of the other units?

5. c equals how many b's? e equals how many b's?

6. Review again and again.

7. Cover the units and review.

1. Show me a line $\frac{1}{2}$ ft. long. How many $\frac{1}{2}$ ft. in the length of each rectangle? how many $\frac{1}{2}$ ft. in the width of each?

2. How many of the rectangles are 1 ft. long? how many are $\frac{1}{2}$ ft. long? how many are 1 ft. wide? how many are $1\frac{1}{2}$ ft. wide?

3. How many 6 in. in the length of each rectangle? how many in the width?

4. Show me the square foot; the square $\frac{1}{2}$ ft. long. A square foot equals how many times a square $\frac{1}{2}$ ft. long?

5. The smaller square is how many inches long? The larger square is how many inches long?

A square 6 in. long equals what part of a square 12 in. long?

6. How many squares 6 in. long in each rectangle?

7. How many squares $\frac{1}{2}$ ft. long in each rectangle?

8. Compare each of the other rectangles with the square foot.

Ex.: A rectangle $\frac{1}{2}$ ft. by 1 ft. equals $\frac{1}{2}$ of a square foot.

9. If we call the length of the small square 1, what is the length of the large square? The square whose length is 1 equals what part of the square whose length is 2?

10. Cover the rectangles and review.

11. Draw a 3-in. square on the blackboard.

12. Compare the 6-in. square with the 3-in. square.

13. The square 6 in. long can be separated into how many squares 3 in. long?

14. Into how many groups of 4 squares 3 in. long can each rectangle be separated?

15. Continue similar work from day to day until the relations are mastered under the different forms of expression.[1]

Review work on relations of quart and pint, pp. 82, 83, 84; and gallon, quart, and pint, pp. 103, 104, Teachers' Book.

Problems. — 1. If a man can earn 50¢ in an hour, in how many hours can he earn 5 times 50¢?

2. A man bought 2 hats at $2 each. He gave the clerk $5. How much change did he receive?

3. Draw rectangles having the ratio of a quart to a pint; of a quart to a gallon.

[1] See remarks under "Freeing the mind from the concrete," Teachers' Book, pp. 19, 20.

4. What is the ratio of a quart to a gallon? $\frac{1}{4}$ is the ratio of what to a gallon? $\frac{1}{2}$ is the ratio of what to a quart?

5. A lady sold a gallon of vinegar and had a quart left. How much had she at first?

6. A man had a jug of molasses. He sold 2 qt. and had 1 pt. left. How many pints did the jug hold? how many quarts?

7. A 12-qt. can holds 3 times as much as a smaller can. How much does the smaller can hold? how many gallons?

8. If 3 qt. of milk sell for 18¢, a gallon will sell for how many thirds of 18¢?

9. 2 equals what part of 4? of 6? of 8?

10 equals how many times 2? Then 10 equals how many times $\frac{1}{2}$ of 4? how many times $\frac{1}{3}$ of 6? $\frac{1}{4}$ of 8?

10. $\frac{5}{2}$ is the ratio of 10 to what? $\frac{5}{3}$ is the ratio of 10 to what? $\frac{5}{4}$?[1]

11. $\frac{3}{5}$ is the ratio of what to 10? $\frac{2}{5}$ is the ratio of what to 10? $\frac{4}{5}$?

12. If a boy can walk $\frac{1}{2}$ mile in 10 min., what

[1] If the pupil hesitates, and the teacher shows him that $\frac{5}{2}$ is the ratio of 10 to 4, the statement is then not the product of his own mind. Let him make the discovery himself. If he cannot do so without taxing him unduly, he is not ready for the question. Engage him in something simpler until power grows. The same spirit which causes children to delight in hunting a thimble or any hidden object in a game will animate them in their work if we adapt it to their condition.

part of the distance can he walk in 6 min.? in 8 min.?

13. At $5¢$ a pencil, how many pencils can be bought for 6 times $5¢$?

14. If 4 fans cost $x¢$, the cost of 6 fans equals how many halves of $x¢$?

15. If 3 oranges sell for $10¢$, 6 oranges will sell for how many $10¢$? 1 doz. oranges will sell for how many $10¢$?

16. Draw a rectangle and call it 10. Draw another whose ratio to the first is $\frac{2}{5}$. What is the ratio of the larger rectangle to the smaller?

If it takes 4 min. to gild the smaller rectangle, how long will it take to gild the larger rectangle?

17. Fred paid a dime for a note book and $\frac{2}{5}$ as much for a sponge. How much did the sponge cost?

18. A slate cost $10¢$ and an eraser $\frac{3}{5}$ as much. How much did the eraser cost? How much less than the slate? The slate cost how much more than the eraser?

19. At the rate of $17¢$ for 3 yd. of ribbon, how many $17¢$ will 6 yd. cost?

20. If 2 melons cost $20¢$, how many 2 melons can I buy for $6 \cdot 20¢$? for $8 \cdot 20¢$?

21. If $\frac{1}{6}$ of a jar of butter is worth $x¢$, how many $x¢$ is the jar of butter worth?

22. If Carrie paid $a¢$ for a doll and $b¢$ for a picture, how much did she pay for both?

23. If the length of a square is 1 ft., what is its perimeter?

24. If the length of a square is x ft., what is its perimeter?

25. What is the perimeter of a rectangle 1 ft. long and $\frac{1}{2}$ ft. wide?

26. James has $4¢$ and Susie has $\frac{3}{4}$ as many. James has how many more than Susie?

27. If 4 girls can sweep a hall in 10 min., 3 girls can sweep what part of it in 10 min.?

28. How many feet in the perimeter of a square $\frac{1}{2}$ ft. long?

29. If 3 horses eat x bushels of oats in a week, how many thirds of x bushels do 4 horses eat in the same time?

30. If 4 pens can be bought for $2¢$, how many can be bought for $3¢$?

31. If 1 ft. of baby ribbon is worth $2¢$, how many $2¢$ is 1 yd. worth? $1\frac{1}{3}$ yd.?

32. What is the ratio of the work which 4 boys can do in an hour to the work which 6 boys can do in an hour?

33. If 6 boys earn $x¢$ in an hour, what part of $x¢$ can 4 boys earn in the same time?

34. What is the ratio of a gallon to a quart? of $\frac{1}{2}$ gallon to $\frac{1}{2}$ quart?

35. What is the relation of 1 yd. to 1 ft.? of $\frac{1}{2}$ yd. to $\frac{1}{2}$ ft.

36. What is the ratio of 5 to 2? of $\frac{1}{2}$ of 5 to $\frac{1}{2}$ of 2?

Place a quarter, a fifty-cent piece, three quarters, and a dollar where they can be observed and handled.

Ratios of Value. — 1. Tell all you can about the relations of these units.

2. If we call the quarter 1, what shall we call each of the other units?

3. If the half dollar is 1, what is each of the other units?

4. If the dollar is 1, each of the other units equals what part of 1?

5. The sum of $\frac{1}{4}$ and $\frac{1}{2}$ equals what? The sum of $\frac{1}{2}$ and $\frac{3}{4}$ equals what?

6. Compare $\frac{1}{4}$ with each of the other units. Compare $\frac{1}{2}$ with each. Compare 1 with each.

7. If there are 25¢ in a quarter, how many 25¢ in each of the other units? How many cents in each?

8. 2 is the relation of what to 25¢? of what to $\$\frac{1}{2}$?

9. $\frac{2}{3}$ is the relation of what to 75¢?

10. $\frac{3}{2}$ is the relation of what to 50¢?

11. Make sentences like this: $\frac{1}{2}$ is the relation of 25¢ to 50¢.

12. Compare 25¢ with each of the others. Compare 50¢ with each. Compare 75¢ with each. Compare $1 with each.

13. If a ball can be bought for 25¢, how many balls can be bought for each of the other units?

14. If $1 be paid for 2 lb. of candy, what part of 2 lb. can be bought for each of the other units?

15. How many 25¢ must be paid for an article whose price is 50¢? whose price is $1? whose price is $2?

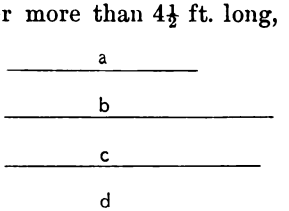
16. How many 50¢ must be paid for an article whose price is $3? whose price is $4?

17. How many dimes will pay for a ball that is worth 50¢?

18. A sled is worth 10 dimes. What piece of money can you give in exchange for the sled?

Draw a line 1 yd. long; another more than $4\frac{1}{2}$ ft. long, but less than 5 ft.; another $1\frac{1}{3}$ yd. long; another $3\frac{1}{2}$ ft. long. Letter as in diagram.

Have the pupil measure with foot rule, while other pupils observe.



Exact Measures.—1. Which of the lines can be exactly measured by a foot? How many feet in *a*? in *c*?

Which lines can be exactly measured by 12 in.? How many 12 in. in *a*? in *c*?

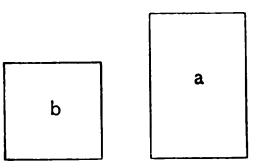
2. Use a 6-in. line as a measure. How many

6 in. lines in a? in c? in d? Is there an exact number of 6-in. lines in b?

3. What is the longest line that will exactly measure a and c?

4. Name different lines that are exact common measures of 1 yd. and of 2 ft.

5. What is the longest line that is an exact common measure of 1 yd. and of 2 ft.?



1. b equals what part of a? In order to compare b with a, into how many parts did you divide b? into how many parts did you divide a?

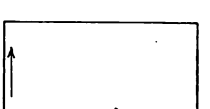
A unit equal to what part of a is an exact measure of each? A unit equal to what part of a is the *greatest* exact measure of each?

A unit equal to what part of b is the greatest exact measure of each?

2. What units are exact measures of 4 and 6? What unit is the greatest exact measure of each?

3. What units are exact measures of 8 and 12? What unit is the greatest exact measure of each?

Dimensions. — 1. Find the lower left corner of the blackboard. From this corner move your hand along an edge.

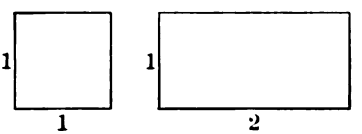


In what direction does this edge extend? Starting from the same corner, move your hand along an edge extending in a different direction.

2. Look at the blackboard. In how many directions do the edges of the blackboard extend from any one of its corners?

3. Find other rectangles, and move your hand so as to show the two dimensions or directions in which they extend.

1. Cut, freehand, rectangles whose dimensions are 1 by 1; that is, whose dimensions are equal.



2. Are the dimensions equal? Measure.

3. Practice cutting and measuring.[1]

4. Cut a larger rectangle whose dimensions are 1 by 1; a smaller one whose dimensions are 1 by 1.

5. Cut rectangles whose dimensions are 1 by 2.

6. Cut larger and smaller rectangles whose dimensions are 1 by 2.

7. Cut rectangles 1 by 3.

1. Draw rectangles whose dimensions are 1 by 1.

2. Draw rectangles whose dimensions are 1 by 2.

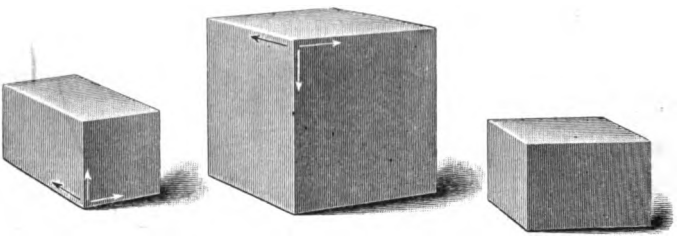
3. Draw rectangles whose dimensions are 1 by 3.

[1] "Quick and correct habits of observation can never be obtained without great practice." — Horace Grant.

"The habit of accurate observation is not taught by nature. It must be acquired by experience." — Payne.

4. Draw rectangles whose dimensions are 2 by 3.

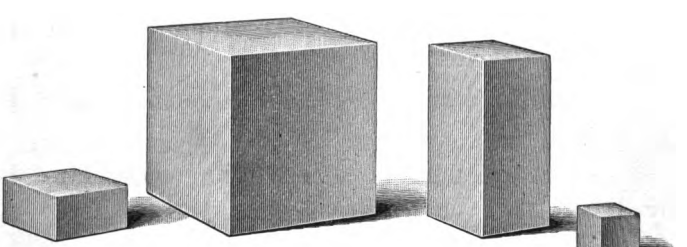
1. Find a corner of one of the solids. How many edges meet in this corner? Move your finger



along each of the three edges. In how many directions do the three edges extend? Look at the solid. How many dimensions has a solid?

2. Show the three dimensions of other solids. In showing the dimensions of a solid, do not start from different corners.

3. Show the three dimensions of a chalk box; of the room.



1. Find solids whose dimensions are 1 by 1 by 1, that is, whose three dimensions are equal.

2. Find solids whose dimensions are 2 by 2 by 1.

3. Find solids whose dimensions are 4 by 2 by 1.

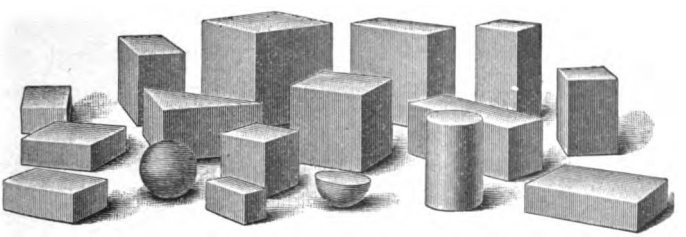
4. Find solids whose dimensions are 3 by 2 by 1.

5. Select solids and tell what their dimensions are.

6. Cover eyes. Tell by sense of touch the dimensions of different solids.

Ex.: This solid is 1 by 1 by 2.

7. Review.



Relative Lengths. — 1. Handle solids and estimate the length of the different dimensions.

Ex.: This solid is 3 in. by 2 in. by 1 in.

After estimating, measure.

2. Close your eyes. Handle solids and tell what you think the dimensions are.

Give many exercises.

3. Select solids and make sentences like this: A solid 3 in. by 1 in. by 1 in. contains 3 cu. in.

4. Find the solid that is 3 in. by 1 in. by 1 in. and the solid that is 3 in. by 2 in. by 1 in.

If we call the first of these solids 1, what is the name of the second? Why should we call the second 2?

Find the solid that we should call 3. What are the dimensions of the solid to which we give the name 3? Find the 4. Find 5. What are its dimensions? Find 6. What are its dimensions?

5. Learn the dimensions of the solids that represent 1, 2, 3, 4, 5, 6.

6. Observe the solids and tell all you can about their relations.

7. How many cubic inches in the smallest of these solids? Of cubic inches build units equal to each of the 6 solids.

8. How many cubic inches equal the unit 1? the 2? the 3? the 4? the 5? the 6? Remove the cubic inches. If we call the smallest solid 3, what should we call each of the others?

9. Find the 12, the 18, the 6, the 15.

10. Tell all you can about the relations of the units 3, 6, 9, 12, 15, 18.

11. The relation of 2 to 1 equals the relation of what to 6?

12. The relation of 3 to 2 equals the relation of what to 6?

13. What is true of the relations of 5 to 6 and 15 to 18? of 6 to 5 and 18 to 15?

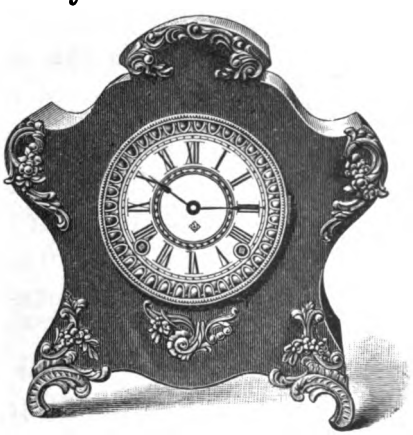
14. What is true of the relations of the units 1,

2, 3, 4, 5, 6 and the units 3, 6, 9, 12, 15, 18? Call the solids 2, 4, 6, 8, 10, 12.

15. What is true of the ratios of 1, 2, 3, 4, 5, 6 and 2, 4, 6, 8, 10, 12?

16. Make sentences like this: The largest cube that can be cut out of the solid 2 in. by 4 in. by 4 in. is 2 in. long.

17. Make sentences like this: Four cubes 2 in. long can be cut out of this solid 4 in. by 4 in. by 2 in.; or 4 2-in. cubes can be cut out of the solid 4 in. by 4 in. by 2 in.



Have pupils sit for 1 or 2 minutes and then write on slips of paper the length of time they think has passed. Tell the length of time. Give other exercises varying the time.

Ratios of Time. — 1. What do we call the time from sunrise to sunrise? How many hours in a

day? In what time does the earth turn round, or rotate? In what part of 24 hr. does it turn half round?

2. That part of the day in which we do not have the light of the sun is called what? In 24 hr. we have day and night, which together make one day.

3. Do we see the sun shine during the entire 24 hr., or day?

4. Did you ever stand in the shadow of a house or a tree on a hot day? Was it cooler in the shadow or in the sunlight? Is it cooler at night or in the daytime? Why? In the night we are in the shadow of what?

1. If the longest line represents 24 hr., what part of 24 hr. does each of the other lines represent?

2. If there are 6 hr. in $\frac{1}{4}$ of a day, how many hours in $\frac{1}{2}$ dy.? in $\frac{3}{4}$ dy.? in 1 dy.?

3. How many hours does each of the lines represent if the longest one represents 24 hr.?

4. Compare 6 hr. with 12 hr.; with 18 hr.; with 1 dy. Compare 12 hr. with each. Compare 18 hr. with each. Compare 1 dy. with each.

5. 2 is the relation of what to 6 hr.? of what to 12 hr.?

6. $\frac{1}{2}$ is the relation of what to 1 dy.?

7. $\frac{1}{4}$ is the relation of what to 1 dy.?

8. 4 is the relation of what to 6 hr.?

9. $\frac{2}{3}$ is the relation of what to 18 hr.?

10. $\frac{3}{4}$ is the relation of what to 1 dy.?

11. If the ratio of the number of school hours to the number of hours in a day is $\frac{1}{4}$, what is the number of school hours?

12. If $\frac{2}{3}$ is the ratio of the distance a man walks to 18 mi., how far does he walk?

13. If $\frac{3}{4}$ is the ratio of the time required to make a journey to 1 dy., what is the time required for the journey?

14. If a man works 6 hr. a day, what part of a day does he work?

15. If 12 hr. is $\frac{2}{3}$ of the time it will take to travel from Chicago to Albany, what is the time?

16. What is the ratio of 24 hr. to 12 hr.? of 1 dy. to 12 hr.?

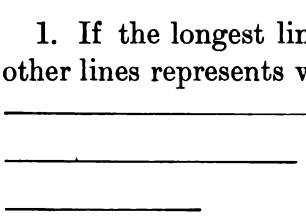
17. Susie spends 6 hr. a day in school. What part of the day is she in school?

If she plays 7 hr., what is the relation of the time she plays to the time she is in school?

18. If 12 mi. equal $\frac{2}{3}$ of the distance between two places, how far are they apart?

19. If a man paints a fence in 18 hr., what part of it does he paint in 6 hr.?

20. What is the ratio of 24 hr. to 6 hr.? What is the ratio of the money a man earns in 24 hr. to the money he earns in 6 hr.? If he earns \$2 in 6 hr., how much will he earn in 24 hr.?



1. If the longest line represents 1, each of the other lines represents what? Compare $\frac{1}{4}$ with the others; $\frac{1}{2}$ with the others; $\frac{3}{4}$ with the others.

2. If the longest line represents 52 wk., or 1 yr., each of the other lines represents what part of 52 wk.?

3. Point to the lines and tell what each represents.

4. Practice drawing groups of lines representing 52 wk.; $\frac{3}{4}$ yr.; $\frac{1}{2}$ yr.; $\frac{1}{4}$ yr. Measure.

5. If the shortest line represents 13 wk., how many 13 wk. does each of the others represent?

6. How many 13 wk. in $\frac{1}{2}$ yr.? in $\frac{3}{4}$ yr.? in 52 wk., or 1 yr.?

7. 13 wk. equal what part of a year? 3 times 13 wk., equal what part of a year?

8. How many weeks are represented by each line?

9. How many 13 wk. in 26 wk.? in 39 wk.? in 52 wk.?

10. Compare 13 wk. with each of the other periods of time.

11. Compare 26 wk. with each. Compare 39 wk. with each. Compare 52 wk. with each. Compare 1 yr. with each.

12. In 1 yr. how many weeks? in $\frac{1}{2}$ yr.? in $\frac{1}{4}$ yr.? in $\frac{3}{4}$ yr.?

13. $\frac{1}{4}$ yr. and $\frac{1}{2}$ yr. equal how many weeks? $\frac{1}{2}$ yr. and $\frac{3}{4}$ yr. equal how many 13 weeks?

14. 2 is the relation of what to 26 wk.?

15. 2 is the relation of what to 13 wk.?

16. $\frac{1}{4}$ is the relation of what to 1 yr.?

17. $\frac{2}{3}$ is the relation of what to 39 wk.?

18. What is the relation of $\frac{1}{2}$ yr. to 39 wk.?

19. Review.

Problems. — 1. Florence attends school 39 wk. What part of a year does she attend?

2. Nellie attended school 13 wk. less than Florence. How many weeks did she attend?

3. Charles is 13 yr. old, and his aunt is twice as old. How old is his aunt?

4. Draw two rectangles having the ratio of 39 to 13.

5. The rent for 13 wk. is $\$x$. What is it for 39 wk.?

6. If $\$x$ be paid for 13 wk. board, how many $\$x$ should be paid for board for 52 wk.? for 1 yr.? for $\frac{2}{2}$ yr.? $\frac{3}{3}$ yr.?

7. How many weeks is it from your sixth birthday until you are 7 yr. old?

Ratios of Time. — 1. What is used to measure the length of your recess?

2. What is used to measure time?

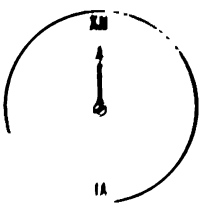
3. Make a drawing of the face of the clock.

4. Tell all you can about the clock.

5. Use a pointer and show the distance the minute hand moves in 1 hr.

6. Show the distance the hour hand moves in 1 hr.

7. Show the distance the hour hand moves in 12 hr: the distance that it moves in a day and a night, or in 1 dy.



8. In turning from 12 to 12, the hour hand measures what part of a day? In turning from 12 to 6, the hour hand measures what part of a day? While the earth is turning once, the hour hand turns from 12 to 12 how many times? How many times does the earth turn in 2 dy.? How many times does the hour hand move around the face of the clock in 2 dy.?

1. If the longest line is x, each of the other lines equals what part of x?

2. If the longest line represents 1 hr., each of the other lines represents what part of an hour?

3. If the longest line represents 60 min., each of the other lines represents what part of 60 min.?

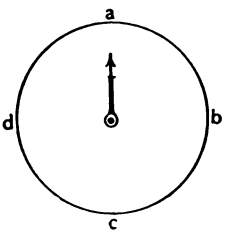
4. If the shortest line represents 15 min., each of the other lines represents how many 15 min.?

5. Point to each of the lines and tell how many minutes it represents.

6. Compare 15 min. with 30 min.; with 45 min.; with 60 min., or 1 hr. Compare 30 min. with each. Compare 1 hr. with each.

7. 15 min. equals what part of $\frac{1}{2}$ hr.? of $\frac{3}{4}$ hr.? of 1 hr.?

8. Indicate on the clock the distance the minute hand moves in 1 hr.; in 60 min.; in 30 min.; in 15 min.; in 45 min.



9. How long does it take the minute hand to move from *a* to *b*? from *a* to *c*? from *a* to *d*? from *a* to *a*?

10. $\frac{1}{4}$ is the relation of what to 1 hr.?

11. 4 is the relation of what to 15 min.?

12. 2 is the relation of what to 30 min.?

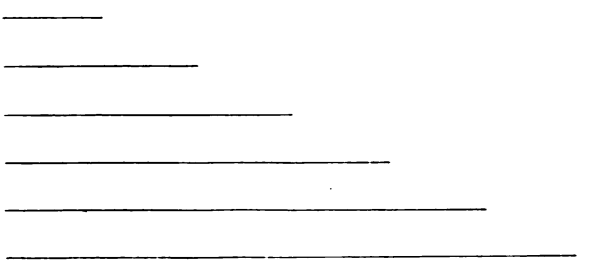
13. $\frac{1}{2}$ is the relation of what to 1 hr.?

14. $\frac{3}{4}$ is the relation of what to 1 hr.?

15. $\frac{2}{3}$ is the relation of what to 45 min.?

16. $\frac{2}{3}$ is the relation of what part of 1 hr. to 45 min.?

1. The longest line represents x. What part of x is represented by each of the other lines?

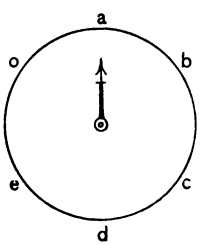


2. The longest line represents 1 hr. What part of an hour is represented by each of the other lines?

3. Point to the different lines and tell the part of an hour that each represents.

4. If the shortest line represents 10 min., each of the other lines represents how many 10 min.? Each line represents how many minutes?

5. Compare 10 min. with the number of minutes represented by each of the other lines. Compare 20 min. with each. Compare 30 min. with each. Compare 40 min. with each. Compare 50 min. with each. Compare 1 hr. with each.



6. How long does it take the minute hand to turn from a to a? What part of an hour does it take the minute hand to turn from a to d? from a to c? from a to e? from a to b? from a to o? from d to o? from e to a? from c to a? from c to b?

7. Make mind pictures of the diagrams, and review again and again.

8. $\frac{1}{2}$ is the relation of how many minutes to 1 hr.?

9. $\frac{2}{3}$ is the relation of how many minutes to $\frac{1}{2}$ hr.?

10. $\frac{3}{2}$ is the relation of what to 40 min.?

11. 6 is the relation of what to 10 min.?

12. $\frac{3}{2}$ is the relation of what to 30 min.?

Problems. — 1. We have 15 min. recess in the morning and 15 min. in the afternoon. What part of an hour is the sum of the recesses?

2. If we write 20 min. and read $\frac{1}{2}$ hr., the time we write equals what part of the time we read?

3. If 40 min. is the time needed to earn $x¢$, how much time is needed to earn $\frac{3}{2}$ $x¢$?

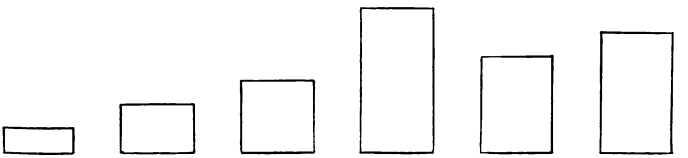
4. At the rate of \$1 in 40 min., how much is earned in 1 hr.?

5. If John can walk from home to the schoolhouse in 10 min., in 1 hr. he can walk how many times this distance?

6. If we sing from 1.30 to 1.45, how many minutes do we sing? what part of an hour? what part of $\frac{1}{2}$ hr.? of 45 min.?

7. John rides x miles in 30 min. At this rate how far does he ride in 45 min.?

8. Lucy walks $\frac{1}{2}$ as fast as Jane. If Jane walks a certain distance in 15 min., how long will it take Lucy to walk the same distance?



Ratios — Cutting. — 1. Use a rule to measure and cut a rectangle 6 in. by 3 in.

2. Cut another 3 in. long and $\frac{1}{2}$ as large as the first.

3. Cut another 3 in. long and $\frac{1}{3}$ as large as the first.

4. Cut another 3 in. wide and $\frac{2}{3}$ as large as the first.

5. Cut another 3 in. long and $\frac{1}{6}$ as large as the first.

6. Cut another 3 in. wide and $\frac{5}{6}$ as large as the first.

7. Arrange the 6 rectangles where you can see them, and give the dimensions of each.

8. Unite different rectangles and make sentences like this: The sum of the rectangle 1 in. by 3 in. and the rectangle 2 in. by 3 in. equals the square rectangle 3 in. long.

9. Tell everything you can about the relations of the rectangles.

10. Make sentences like this: A 2-in. square is the largest square that can be cut out of a rectangle 2 in. by 3 in.

11. Make sentences like this: In a rectangle 2 in. by 3 in. there is a 2-in. square and a rectangle 1 in. by 2 in.

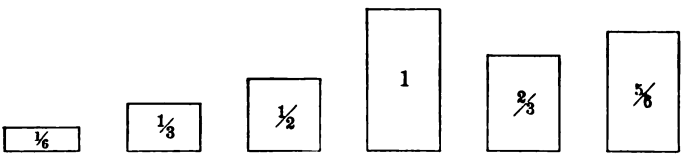
12. Make mind pictures and tell everything you can about these rectangles.

13. Call the rectangles a, b, c, d, e, o; a being the smallest.

14. Suppose that each of these pieces of paper represents a mat, and that o costs $\$x$. What, then, equals the cost of each of the other mats?

15. If d costs $\$x$, what equals the cost of each of the others?

16. Pupils question one another.



Ratios. — 1. If the largest rectangle is 1, each of the other rectangles equals what part of 1?

2. Point to the $\frac{1}{2}$; the $\frac{1}{3}$; the $\frac{2}{3}$; the $\frac{1}{6}$; the $\frac{5}{6}$.

3. Point and tell the part represented by each again.

4. Compare each part with every other part.

5. How many square inches in the smallest of the rectangles? Show me one row of 3 sq. in. How many rows of 3 sq. in. in each rectangle?

6. If we call the smallest rectangle 3, what is the number name of each of the other rectangles? Show me the 18; the 9; the 6; the 12; the 3; the 15.

7. Compare 3 with each of the others. Compare 6 with each. Compare 9 with each. Compare 12 with each. Compare 15 with each. Compare 18 with each.

8. Which of the units can be exactly measured by 6?

9. Which unit is 4 times as large as the unit 3?

10. Which unit is 3 times as large as $\frac{1}{2}$ of 6?

11. 4 is the relation of what to 3?

12. 3 is the relation of what to 6?

13. What is the relation of 12 to 6?

14. The relation of 12 to 6 equals the relation of what to 4?[1]

15. The relation of 18 to 9 equals the relation of what to 3?

16. Make sentences like this: The relation of 3 to 6 equals the relation of 5 to 10.

17. $\frac{1}{3}$ of 9 equals what part of 18? of 12? of 15?

18. A unit equal to what part of 15 must be added to 15 to make 18? $\frac{1}{5}$ of 15 equals what part of 18?

19. 18 equals how many fifths of 15? how many thirds of 9?

20. 3 is the ratio of what to $\frac{1}{2}$?

1. If one package of seeds is needed to plant a garden containing x sq. yd., how many such packages are needed to plant a garden containing $1\frac{1}{2}$ x sq. yd.?

2. If it takes an hour to gild the rectangle $\frac{1}{2}$, how long will it take to gild the rectangle $\frac{2}{3}$?

3. What is the ratio of 1 to $\frac{1}{3}$? If 6 ft. equals $\frac{1}{3}$ of the length of a sidewalk, how long is the sidewalk?

[1] "Each increment of intellectual progress implies an increase in representativeness of thought." "The universal process of intelligence is the assimilation of impressions." — Herbert Spencer.

4. Mary and Edna bought a cake for 6¢. If each paid 3¢, what part of the cake ought each to have? If Mary paid 4¢ and Edna 2¢, what part of the cake ought each to have?

5. Sabra planted pinks in $\frac{1}{3}$ of her flower bed; pansies in $\frac{1}{2}$ of it. What part of the bed remained unplanted? The space unplanted equals what part of the space given to the pinks? What part of the space given to the pansies? How many more sixths of the bed were given to pansies than to pinks?

6. If $\frac{2}{3}$ of the cider in a jar is worth 12¢, what is all of it worth?

7. What is the ratio of the amount you can buy for 1¢ to the amount you can buy for $\frac{1}{2}$¢?

At $\frac{1}{2}$¢ each, how many caramels can you buy for 9¢?

8. Mr. Smith earns $15 a week and spends $9. How much does he save in 1 wk.? In how many weeks does he save $18?

9. A man earns $5 as often as a boy earns $3; while the boy earns $6, how many $5 does the man earn?

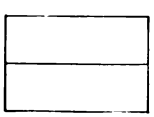
10. Cut a rectangle; cut another 3 times as large as $\frac{1}{2}$ of the first. If the first is 2, what is the second? If the first is 1, what is the second? If the second is 1, what is the first? If the second is x, the first equals what part of x?

11. Cut a rectangle; cut another 5 times as large as $\frac{1}{2}$ of the first. Give different number names to

the first and tell what the name of the second is. Give different names to the second and tell what the first should be. *Ex.:* If the second is 7, the first equals $\frac{2}{5}$ of 7.

12. Cut a rectangle and call it 4. Cut a rectangle equal to $\frac{3}{2}$ of 4. What is the name of the second rectangle cut?

1. Find $\frac{1}{2}$ of this rectangle; the $\frac{2}{2}$; the $\frac{3}{3}$; the $\frac{2}{3}$; the $\frac{1}{3}$; the $\frac{1}{6}$; the $\frac{5}{6}$.



2. Point to $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{6}{6}$, or 1.

3. Tell all the relations you can.

4. Compare each equal part with every other equal part. *Ex.:* $\frac{1}{6}$ equals $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{2}$, $\frac{1}{4}$ of $\frac{2}{3}$, $\frac{1}{5}$ of $\frac{5}{6}$, $\frac{1}{6}$ of 1.

1. What is the ratio of $\frac{3}{3}$ to $\frac{1}{2}$ of $\frac{2}{3}$? of 1 to $\frac{2}{3}$? When two rectangles have the ratio of 1 to $\frac{2}{3}$, the larger equals how many halves of the smaller? the smaller equals how many thirds of the larger?

2. A cloth 4 ft. square will cover $\frac{2}{3}$ of the table. A unit equal to what part of this cloth must be added to cover the table?

3. If you plant potatoes in $\frac{1}{2}$ your garden and peas in $\frac{1}{3}$ of it, how much more of the garden do you use for potatoes than for peas?

4. A cloak cost \$6 and a hat $\frac{2}{3}$ as much. How much did the hat cost?

5. What is the ratio of the amount of sugar you can buy for \$6 to the amount you can buy for \$4?

6. $\frac{2}{3}$ of a man's crop is corn and $\frac{1}{6}$ wheat. The ratio of the number of bushels of wheat to the number of bushels of corn is what?

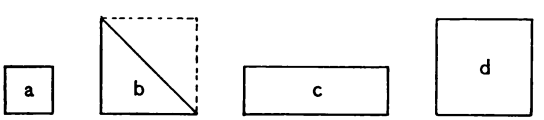
7. Harry has 6¢. He gives $\frac{2}{3}$ of his money for marbles and $\frac{1}{6}$ for candy. How much has he left?

8. Mr. Jones owns $\frac{1}{3}$ of a 12-acre field and Mr. Smith owns the rest. How many acres belong to Mr. Smith?

9. A boy works $\frac{1}{2}$ hr. and plays $\frac{1}{3}$ hr. What is the ratio of the time he works to the time he plays? $\frac{1}{2}$ is the ratio of the time he works to what? The ratio of the time he plays to 1 hr. is what?

10. $\frac{1}{6}$ of my marbles are black and $\frac{1}{2}$ white. The number of black equals what part of the number of white ones?

11. Walter had 6¢ and spent a nickel for car fare. What part of his money did he spend? 6¢ is how much more than a nickel? A nickel is how much less than 6¢?



Draw these units on the blackboard, making *a* 1 ft. long.

Ratios. — 1. Tell all you can about these units.

2. Tell length and width in feet; in half feet; in 6 in.

3. Tell dimensions in yards. *Ex.:* a is a square $\frac{1}{3}$ of a yard long.

4. The length of triangle b equals what part of a yard? Its height equals what part of a yard?

5. What is the length in feet of the perimeter of a? of c? of d?

6. What is the perimeter of a in yards? of c? of d?

7. Tell all you can about the relations of these units without observing them.

8. Triangle b equals what part of d? $\frac{2}{3}$ of c equals what part of d? Triangle b equals what part of c?

9. The unit a equals what part of b? of c? of d?

10. What is the relation of triangle b to each of the other units? of c to each? of d to each?

11. $\frac{1}{2}$ of a equals what part of b? of c? of d?

12. If a were a lot containing 4 acres, how many acres would there be in b? in c? in d?

13. If d is worth \$1000, what part of \$1000 is each of the other lots worth?

14. If triangle b is worth \$$x$, what is each of the other units worth?

1. 2 is the relation of which unit to a? of which to b?

2. $\frac{2}{3}$ is the relation of which unit to c?

3. $\frac{1}{2}$ is the relation of which unit to d?

4. Which unit is 4 times as large as $\frac{1}{3}$ of c?

5. $\frac{4}{3}$ is the relation of which unit to c?

6. The relation of b to a equals the relation of what to b?

7. What is the relation of b to a? of d to b?

8. Review.

Problems.—1. Annie has pink cardboard enough to make 4 cards each the size of b, and blue enough to make 4 each the size of d. The pink cardboard equals what part of the blue?

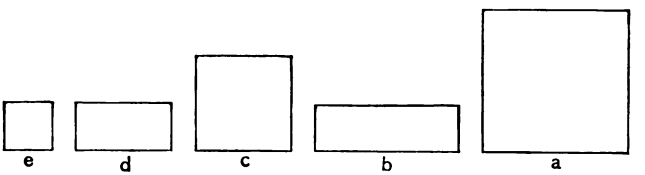
2. What is the ratio of the number of roses that can be raised in a garden equal to c, to the number that can be raised in a garden equal to d? If 39 roses can be raised in c, how many can be raised in d?

3. If d were a napkin, how would you fold it to make it the shape of b?

4. Monday a carpenter painted a shelf 4 times as long as a; Tuesday, one as wide as a and 4 times as long as c. By this work he earned how many times as much Tuesday as Monday? Painting a shelf equal to $\frac{1}{2}$ of a requires what part of the amount of paint needed for painting a shelf equal to $\frac{1}{2}$ of c?[1]

[1] If pupils do not readily make the inferences required, let them represent the units by drawing. By activity in observing, doing, and imaging the mind is gradually freed from the need of the visible representation.

5. If I use a bottle of gilding in coloring 30 cards each the size of *d*, how much will be used in coloring 30 cards each the size of *a*? In coloring 30 cards each the size of *c*?



Ratios — Square Yard. — 1.[1] Draw these units on the blackboard, making *e* 1 ft. long.

2. Tell all you can about these units.

3. Make comparisons.

4. Draw a line a yard long. How many of the figures are 1 yd. long? how many $\frac{2}{3}$ yd. long?

5. Each unit is how many feet long? how many 12 in. long? how many 6 in. long?

6. Give the dimensions of each in yards or parts of yards. *Ex.:* *e* is a square $\frac{1}{3}$ of a yard long; *d* is a rectangle $\frac{1}{3}$ yd. by $\frac{2}{3}$ yd.

7. Give the dimensions of each in feet.

8. Give the dimensions of each in 12 in.

Ex.: *b* is 12 in. by 3·12 in.

9. Give the dimensions in 6 in.

Ex.: *b* is 2·6 in. by 6·6 in.

[1] To talk about that which is not in the mind clogs rather than facilitates mental action.

"Without an accurate acquaintance with the visible and tangible properties of things, our conceptions must be erroneous, our inferences fallacious, and our operations unsuccessful."

10. What is the perimeter of each in yards?
Ex.: $1\frac{1}{3}$ yd. is the perimeter of *e*.

11. Give the number of feet in the perimeter of each.

12. Give the number of 12 in. in the perimeter of each.

13. Give the number of 6 in. in the perimeter of each.

14. Review without observing the units.

1. If *a* represents the top of a table, what must be the dimensions of a cloth which will exactly cover it?

2. What is the ratio of a yard to a foot? of 3 yd. to 1 ft.? What is the cost of a fish line 3 yd. long at 2¢ a foot?

3. What is the ratio of the width of a door 1 yd. wide to the width of a window 4 ft. wide?

4. The length of a newspaper is 24 in. This equals $\frac{4}{3}$ of its width. What is its width? Its width equals what part of a yard? Its length equals what part of a yard? How many feet? What is the perimeter of the newspaper?

5. The perimeter of a square envelope is 24 in. What is its length?

6. The perimeter of a square envelope is 4 *a*. What is its length?

7. The width of a square envelope is *b*. What is its perimeter? What is the ratio of the perimeter to the length? of the length to the perimeter?

8. A sheet of note paper is 9 in. long. Its length equals what part of a foot? What part of a yard? $\frac{3}{4}$ of 1 ft. equals what part of 3 ft.?

9. A hall is 12 ft. long. How many yards long must a strip of carpet be to reach from one end of the hall to the other? At 50¢ a yard, what is the cost of this strip?

10. A sewing-table is 1 yd. long and a stand 24 in. How much longer is the table than the stand? The length of the table equals how many $\frac{1}{2}$'s of the length of the stand?

Problems. — 1. A man has 6 qt. of milk. He sells $\frac{1}{2}$ of it at 4¢ a pint. How much does he receive?

2. At 27¢ a peck, how many 27¢ will 1 bu. of potatoes cost?

If the pupils have not clearly defined ideas of the relative size of the bushel, peck, quart, etc., return to the things from which such ideas are gained. Correct reasoning is impossible when the relation upon which the inference rests is not in the mind. Not progress, but vicious mental habits will result from leading pupils to say what they do not see.

3. A man had seven $100 bills. He paid two of them for a team. How many had he left?

4. If a man sleeps 8 hr. a day, how many hours does he sleep in 4 dy.?

5. If a barrel of flour last a family 5 wk., how many bbl. will last them 20 wk.?

6. If you go to school 5 dy. each week, how many days do you attend in 4 wk.?

7. If a man earns $10 a week, how much does he earn in 4 wk.?

8. How many yards long must a strip of wall paper be to reach from ceiling to floor of a room 10 ft. high?

9. How many yards long must a curtain be for a doorway 6 ft. in height? If the door is wide enough to require two widths, how many yards must you buy?[1]

1. At $6\frac{1}{2}¢$ a yard, what does 4 yd. of curtain goods cost?

2. At $1 a peck what does 1 bu. of clover seed cost?

3. If $\frac{1}{2}$ doz. bananas can be bought for $10¢$, how many bananas can be bought for $15¢$?

4. At the rate of 3 pears for $10¢$, how many $10¢$ does a dozen pears cost? How many cents?

5. At $60¢$ a doz., how many oranges can you buy for $20¢$? What is the ratio of 20 to 60? $\frac{1}{3}$ is the ratio of what to 12?

6. $13 is $\frac{1}{2}$ of the cost of a cow. What is the cost?

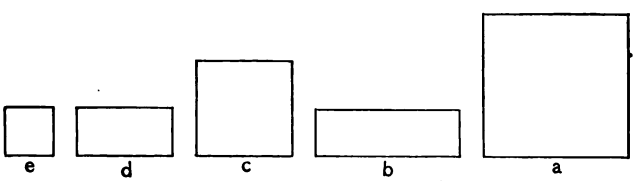
[1] Let pupils discover such facts for themselves. Encourage them to make problems suggested by their own observation of things about them. Do not interfere with the free action of the mind by too much dictation or questioning.

7. 3·52 wk. is the time Alice lived in New York. How many years did she live there?

8. If the ratio of one card to another is $\frac{2}{3}$, the larger card equals how many halves of the smaller? If we call the smaller card 4, what shall we call the larger? if we call the smaller card 8? if we call it 16?

9. I think of two lines: one is 36 in. long and the other 2 ft. How much must be added to the shorter line to make it equal the longer?

10. The perimeter of a platform 3 ft. in width is 22 ft. How long is it?



Ratios — Square Yard. — 1. Unite units and tell what the sum equals.

2. The sum of e and d equals what?

3. The sum of e and b equals what?

4. The sum of b, d, and e equals what part of a?

5. The sum of b, c, and d equals what?

6. The unit b equals what part of a?

7. How many square feet in b? How many rows of 3 sq. ft. in a? How many square feet in a? a is a square yard. How many square feet in a square yard?

8. A square foot equals what part of a square yard?

9. How many square feet in c? The unit c equals what part of the square yard?

10. The square $\frac{2}{3}$ yd. long equals what part of a square yard?

11. A rectangle 1 ft. by 3 ft. equals what part of a square yard?

12. A rectangle $\frac{1}{3}$ yd. by 1 yd. equals what part of a square yard?

13. How many yards in the perimeter of a square yard? how many feet? how many 12 in.?

Problems. — 1. A man pays all of his money for 9 lots. What part of his money does he pay for 3 of them?

2. I gave each of 2 boys \$3, and had left $\frac{1}{2}$ as much as I gave away. How much had I at first?

3. A pavement is made of stones 1 yd. square. How many yards in the perimeter of one of these stones? how many 3 ft.? how many feet?

4. If it takes 8 min. to paint a table 2 ft. by 3 ft., how long does it take to paint a table 1 yd. by 1 yd.?

5. How many yards of fringe are required for the ends of a scarf $\frac{2}{3}$ yd. in width?

6. A rug containing 1 sq. yd. will cover how many times as great a space as one 1 yd. long and $\frac{2}{3}$ yd. wide?

7. If $30¢$ is paid for 1 sq. yd. of oilcloth, what should be paid for a strip 1 yd. long and 1 ft. wide?

8. The cost of a yard of wire is how many times the cost of a foot?

9. The cost of 36 in. of wire is how many times the cost of 12 in.?

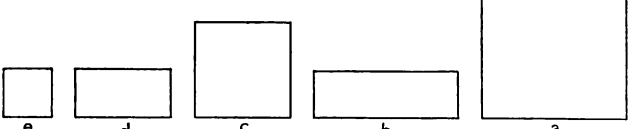
10. If 12 yd. of carpet cost $\$x$, 18 yd. cost how many halves of $\$x$? 20 yd. cost how many thirds of $\$x$?

11. If 14 bu. of apples cost $\$y$, 21 bu. cost what? 18 bu.?

12. If 3 sq. ft. of bristol board cost $a¢$, what is the cost of 1 sq. yd.?

13. What is the ratio of $\frac{1}{3}$ to $\frac{1}{9}$? of $\frac{2}{3}$ to $\frac{2}{9}$? of $\frac{1}{9}$ of a square yard to $\frac{1}{3}$ of a square yard?

14. If $x¢$ is the cost of plastering a square foot, what is the cost of plastering a square yard?



Ratios. — 1. $\frac{1}{2}$ of e equals what part of $\frac{1}{2}$ of d?

2. e equals what part of b?

3. $\frac{1}{2}$ of e equals what part of $\frac{1}{2}$ of b?

4. Compare $\frac{1}{2}$ of c with $\frac{1}{2}$ of each of the other units.

5. If the length of e is 1, what is the length of c? of a?

6. Point to the 1; the 2; the 3. Point to the square of the 1; of the 2; of the 3.

7. The square of the 1 equals what part of the square of the 2?

8. The square of the 1 equals what part of the square of the 3?

9. The square of 2 equals how many times the square of 1?

10. The square of 2 equals what part of the square of 3?

11. The square of 3 equals how many times the square of 1?

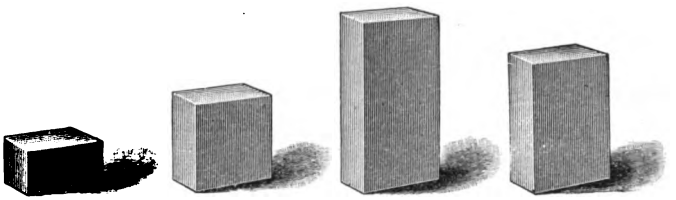
12. Show me $\frac{1}{4}$ of the square of 2. The square of 3 is how many times as large as $\frac{1}{4}$ of the square of 2? The square of 3 equals how many fourths of the square of 2?

13. Draw a 6-in. line on the blackboard and call the line 1; draw a 2; draw a 3. Draw the square of the 1; of the 2; of the 3. Compare each of the squares with the other two.

14. What is the ratio of a sq. yd. to a sq. ft.? of 2 sq. yd. to 2 sq. ft.? of $\frac{1}{2}$ sq. yd. to $\frac{1}{2}$ sq. ft.?

15. 2 is the ratio of a to b. What is the ratio of $3a$ to $3b$?

Ratios of Corresponding Parts. — 1. Compare a part of one unit with a corresponding part of another.



Ex.: Compare $\frac{b}{2}$ with $\frac{c}{2}$ (read, $\frac{1}{2}$ of b with $\frac{1}{2}$ of c). Compare $\frac{d}{2}$ with $\frac{b}{2}$.

2. a = what part of b? $\frac{a}{2}$ = what part of $\frac{b}{2}$?[1]

3. a = what part of c? $\frac{a}{2}$ = what part of $\frac{c}{2}$?

4. a = what part of d? $\frac{a}{2}$ = what part of $\frac{d}{2}$?

5. b = how many times a? $\frac{b}{2}$ = how many times $\frac{a}{2}$?

6. b = what part of c? $\frac{b}{2}$ = what part of $\frac{c}{2}$?

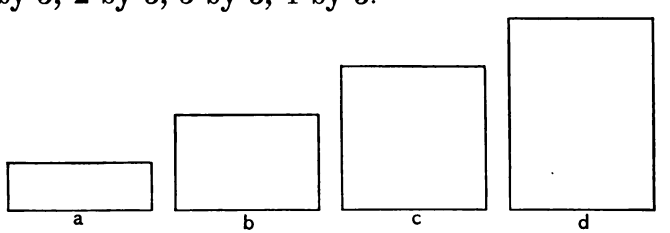
[1] Try to secure the observing, the thinking, which will lead to correct expression. When a pupil says, "a is $\frac{1}{2}$ of b," he should be incited to closer observing. Is a any part of b?

7. b = what part of d? $\frac{b}{2}$ = what part of $\frac{d}{2}$?

8. c = how many times a? $\frac{c}{2}$ = how many times $\frac{a}{2}$?

9. c = how many times $\frac{b}{2}$? $\frac{c}{2}$ = how many times $\frac{1}{2}$ of $\frac{b}{2}$?

1. Give each pupil the following rectangles: 1 by 3, 2 by 3, 3 by 3, 4 by 3.



2. Observe and compare corresponding parts.

3. Cover $\frac{2}{3}$ of a and $\frac{2}{3}$ of b. Compare $\frac{a}{3}$ with $\frac{b}{3}$.

4. a = what part of c? $\frac{a}{3}$ = what part of $\frac{c}{3}$?

5. a = what part of d? $\frac{a}{3}$ = what part of $\frac{d}{3}$?

6. Compare $\frac{b}{3}$ with $\frac{1}{3}$ of each of the other units.

7. Compare $\frac{b}{2}$ with $\frac{1}{2}$ of each of the other units.

8. Compare $\frac{c}{3}$ with $\frac{1}{3}$ of each of the other units.

9. Review.

Problems. — 1. If a weighs 8 oz. (see p. 56), what is the weight of $\frac{1}{2}$ of a? What is the weight of $\frac{1}{2}$ of each of the others?

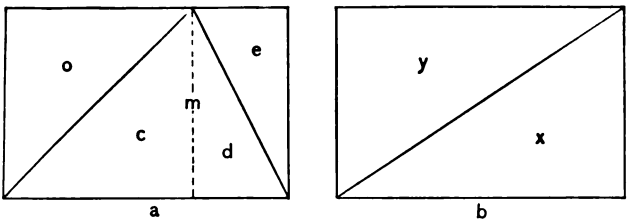
2. Willie earns 20¢ an hour and Harry 10¢. In $\frac{1}{2}$ hr. Willie earns how many times as much as Harry in $\frac{1}{2}$ hr.?

3. If for a large box of candy I pay 4 times as much as for a small box, for $\frac{1}{2}$ of the large box I pay how many times as much as for $\frac{1}{2}$ of the small box?

4. A 12-ft. line is how many times as long as a 6-ft. line? $\frac{1}{3}$ of a 12-ft. line is how many times as long as $\frac{1}{3}$ of a 6-ft. line?

5. 12 yd. are required for a dress for Mrs. Keith and 4 yd. for her little girl. At the same price per yd. the cost of the little girl's dress equals what part of the cost of her mamma's dress? The cost of $\frac{1}{2}$ of the little girl's dress equals what part of the cost of $\frac{1}{2}$ of her mamma's dress?

6. If black ribbon costs 10¢ a yard and blue 6¢ a yard, what is the ratio of the cost of $\frac{1}{2}$ yd. of black ribbon to the cost of $\frac{1}{2}$ yd. of blue ribbon? What is the ratio of the cost of 5 yd. of black ribbon to the cost of 5 yd. of blue ribbon?



Draw enlarged diagram on blackboard.

Ratios of Triangles and Rectangles. — 1. Find all the equals that you can.

2. a = what? d = what? c = what?

3. The sum of c and d = what? The sum of c and d = what part of rectangle a?

4. The sum of e and o = what part of rectangle a?

5. The sum of c and d = the sum of what two triangles?

6. Triangle m is the sum of what two triangles?

7. Triangle m = the sum of what two triangles?

8. Triangle m = what triangle?

9. Triangle m = what other triangle?

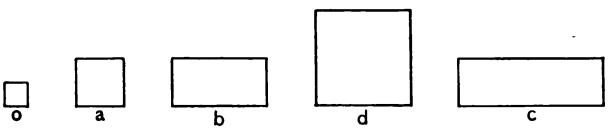
10. Why do you think triangle m = triangle x?

11. What is true of rectangles a and b? Triangle m = what part of a? x = what part of b? Why are triangles m and x equal?

12. The number of square feet in triangle m = what part of the number of square feet in rectangle b?

13. If triangle *m* is worth 5¢, what is rectangle *b* worth?

14. Review without observing the diagram.



Ratios. — 1. If *d* is 1, to what part of 1 is each of the other units equal? Show me the $\frac{1}{4}$; the $\frac{1}{2}$; the $\frac{3}{4}$; the $\frac{4}{4}$, or 1.

2. Make sentences like this: The sum of $\frac{1}{2}$ and $\frac{1}{4} = \frac{3}{4}$.

3. Make sentences like this: $\frac{3}{4}$ less $\frac{1}{2} = \frac{1}{4}$.

4. What part of $\frac{1}{2}$ is as large as $\frac{1}{4}$? What part of $\frac{3}{4}$ is as large as $\frac{1}{4}$? What part of $\frac{3}{4}$ is as large as $\frac{1}{2}$?

5. $\frac{1}{2}$ is how many times as large as $\frac{1}{4}$? $\frac{3}{4}$ is how many times as large as $\frac{1}{4}$? 1 is how many times as large as $\frac{1}{4}$?

6. Show me $\frac{1}{2}$ of the $\frac{1}{2}$. $\frac{3}{4}$ is how many times as large as $\frac{1}{2}$ of the $\frac{1}{2}$? 1 is how many times as large as $\frac{1}{2}$ of $\frac{1}{2}$?

7. Show me $\frac{1}{3}$ of the $\frac{3}{4}$. 1 is how many times as large as $\frac{1}{3}$ of $\frac{3}{4}$?

8. Compare $\frac{1}{4}$ with each of the other units; compare $\frac{1}{2}$ with each; compare $\frac{3}{4}$ with each; compare 1 with each.

1. 4 is the relation of which unit to o? of which unit to a?

2. What is the relation of o to a? of a to d?

3. The relation of o to a equals the relation of a to what?

4. What is the relation of each unit to o?

5. What is the relation of each unit to a? of each unit to $\frac{1}{3}$ of c?

6. 8 is the relation of which unit to o? 12 is the relation of which unit to o? 16 is the relation of which unit to o?

7. c is how many times as large as $\frac{1}{2}$ of b?

8. $\frac{3}{2}$ is the relation of which unit to b?

1. Show $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{4}$ of the blackboard. Compare $\frac{1}{4}$ of the blackboard with the other parts. Compare $\frac{1}{2}$ with the other parts. Compare $\frac{3}{4}$ with the other parts. Compare the entire board with the different parts.

2. The sum of $\frac{1}{4}$ and $\frac{1}{2}$ of the blackboard equals what part of the blackboard?

3. The sum of $\frac{1}{2}$ and $\frac{1}{4}$ of 1 yr. equals what part of 1 yr.?

4. The sum of $\frac{1}{2}$ and $\frac{1}{4}$ of 52 wk. equals what part of 52 wk.?

5. The sum of $\frac{1}{2}$ and $\frac{1}{4}$ of 1 yd. equals what part of 1 yd.?

6. The sum of $\frac{1}{2}$ and $\frac{1}{4}$ of 36 in. equals what part of 36 in.?

7. The sum of $\frac{1}{2}$ and $\frac{1}{4}$ of x = what part of x?

8. How many squares the size of o in the $\frac{1}{4}$? (see p. 60), in the $\frac{1}{2}$? in the $\frac{3}{4}$? in the 1?

9. Draw enlarged units on the blackboard and review.

10. Cover the units. Image them. Review.

1. Find solids representing $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1, and tell what each represents. *Ex.:* This represents $\frac{1}{4}$.

2. Compare each unit with the others.

3. Make sentences like this: If the $\frac{1}{4}$ weighs 3 oz., the $\frac{1}{2}$ weighs 6 oz.

4. What is true of the relations of $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 and of the relations of 1, 2, 3, and 4?

5. If the unit 1 is separated into 24 equal parts, the unit $\frac{1}{4}$ equals how many of these parts? the unit $\frac{1}{2}$ equals how many? $\frac{3}{4}$? $\frac{4}{4}$?

6. 2 is the relation of what unit to $\frac{1}{4}$? of what unit to $\frac{1}{2}$?

7. 4 is the relation of what unit to $\frac{1}{4}$?

8. $\frac{3}{2}$ is the relation of what unit to $\frac{1}{2}$?

9. Handle and tell dimensions of units with eyes closed.

10. Give two units. Pupils tell relation with eyes closed.

11. Tell relative cost with eyes closed.

12. Tell relative weight with eyes closed.

Cutting and Drawing. — Cut sets of rectangles of different sizes representing $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1. Compare the units in each set.

Have one pupil draw *one* of a set of units on the blackboard, and give its name. Other pupils draw entire set, using the one as the standard.

Cut triangles representing 1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$.

Problems. — 1. A man paid $\$\frac{1}{2}$ for dinner and $\$\frac{1}{4}$ for lunch. How much did he pay for both?

2. If $\frac{3}{4}$ yd. of lace cost $15¢$, 1 yd. costs how many thirds of $15¢$? how many cents?

3. The cost of $\frac{1}{2}$ yd. equals what part of the cost of $\frac{3}{4}$ yd?

4. $\frac{3}{4}$ of a dozen is how many more than $\frac{2}{3}$ of a dozen?

5. I think of a line 1 ft. in length and of a line $\frac{1}{3}$ as long. How many inches in each line?

6. At $x¢$ for $\frac{1}{2}$ lb. of tea, what part of $x¢$ does $\frac{1}{4}$ lb. cost?

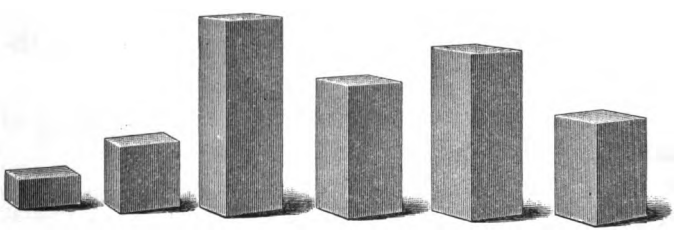
7. $\frac{2}{3}$ of the roses in a bouquet are red and $\frac{1}{6}$ white. The white roses equal what part of the red? If the remaining roses were pink, the pink equal what part of the bouquet?

8. $\frac{1}{2}$ gal. less $\frac{1}{3}$ gal. equals what part of 1 gal.? $\frac{1}{2}$ gal. is how much more than $\frac{1}{3}$ gal.? than $\frac{1}{4}$ gal.?

9. Warren painted $\frac{1}{4}$ of a wall yesterday and $\frac{1}{3}$ of the remainder to-day. How much is still to be painted?

10. The sum of $\frac{1}{3}$ and $\frac{1}{2}$ ft. equals what part of 1 ft.? The sum of $\frac{1}{3}$ and $\frac{1}{2}$ yd. equals what part of 1 yd.? The sum of $\frac{1}{3}$ and $\frac{1}{2}$ of a equals what part of a?

11. Draw three lines, making the second 3 times the first and the third 3 times the second. The shortest line equals what part of each of the other lines? $\frac{1}{2}$ and $\frac{1}{4}$ of each line equal what part of the line? $\frac{1}{3}$ and $\frac{1}{2}$ of each line? $\frac{1}{3}$ and $\frac{2}{9}$ of each line? Show me $\frac{3}{3}$ of each line; $\frac{2}{2}$; $\frac{4}{4}$; $\frac{8}{8}$.



Ratios. — 1. Place solids, having the ratios of those given above, where they can be handled.

2. If d is 1, what is each of the other solids? Point to the $1\frac{1}{2}$; to the $1\frac{1}{4}$; to the $\frac{4}{4}$, or 1; to the $\frac{3}{4}$; to the $\frac{1}{2}$; to the $\frac{1}{4}$.

3. Name the units in order, beginning with the $\frac{1}{4}$.

4. Name them in order, beginning with the $1\frac{1}{2}$.

5. Name them in order without looking at them.

6. Compare each unit with each of the other units.

7. Have pupils close eyes and tell, by handling, the name of the unit.

8. Close eyes; handle and compare units.

9. If c is 1, what is e? What is each of the others?

10. Handle units and tell the name of each.

11. Show me $\frac{1}{2}$ of the door; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{6}{6}$.

12. Close eyes, handle solids, and tell relative size, weight, and cost.

13. Cut sets of rectangles having the same relation as the solids above. Compare the rectangles.

14. Draw groups of lines having the same relations. Compare.

15. Let a pupil draw a unit on the blackboard and give its name; let another draw a related unit. *Ex.:* This is $\frac{1}{3}$; draw $\frac{1}{2}$.

16. Make sentences like this: If $\frac{1}{2}$ costs 17¢, $\frac{1}{6}$ costs $\frac{1}{3}$ of 17¢.

1. If $\frac{1}{4}$ weighs 5 lb., what does $\frac{1}{2}$ weigh? If $\frac{1}{2}$ weighs 10 lb., how much does $\frac{1}{6}$ weigh?

2. If $\frac{5}{6}$ of a yard of velvet costs 70¢, 1 yd. will cost how many fifths of 70¢.

3. Walter lives $\frac{1}{2}$ mi. from the schoolhouse. When he has traveled $\frac{1}{3}$ of the distance, what part of a mile must he still travel?

4. If 3 tons of coal cost $18, what does 1 ton cost? what does $\frac{1}{3}$ of a ton cost?

5. If $\frac{1}{4}$ bu. of corn costs 10¢, what does $\frac{1}{2}$ bu. cost? what do 3 pk. cost?

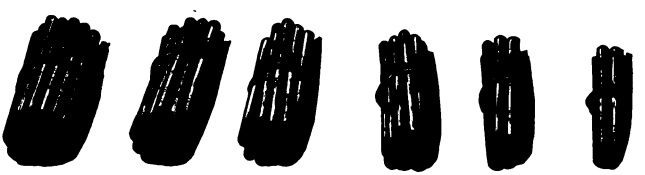
6. The cost of 1 qt. equals what part of the cost of 1 gal.? The cost of $\frac{1}{2}$ qt. equals what part of the cost of $\frac{1}{2}$ gal.?

7. A dress for a doll requires $\frac{2}{3}$ yd. and a skirt $\frac{1}{2}$ yd. If they are made of the same material the cost of the skirt equals what part of the cost of the dress?

8. Sarah had $\frac{3}{4}$ yd. of ribbon. She gave $\frac{1}{2}$ yd. to Jane. How much had she left?

9. Arthur carried a package containing $\frac{1}{2}$ lb. of coffee, $\frac{1}{4}$ lb. of tea, and $\frac{1}{4}$ lb. of cheese. How much did the package weigh?

10. A boy added $\frac{1}{3}$ yd. to 1 yd., and then erased $\frac{1}{4}$ of the line. How long was the line then?



Give each pupil sticks tied into bundles of 4; 2 · 4's; 3 · 4's; 4 · 4's; 5 · 4's; and 6 · 4's.

Ratios. — 1. If we call the largest bundle 1, what name shall we give to each of the others?

2. Show me the $\frac{1}{2}$; the $\frac{1}{3}$; the $\frac{2}{3}$; the $\frac{1}{6}$; the $\frac{5}{6}$; the $\frac{6}{6}$.

3. Find the unit $\frac{1}{6}$. How many equal parts do you see in this unit? How many 4 parts do you find in each of the units?

4. If we call the smallest unit 4, what name shall we give to each of the other units?

5. Name the units in order, beginning with the 4. Repeat in order, beginning with the largest.

6. Make sentences like this: The sum of 4 and $8 = 12$.

7. Make sentences like this: 20 less $12 = 8$.

8. Make sentences like this: I see that $\frac{4}{5}$ of $20 = 16$.

9. Compare each unit with each of the others.

10. Show the number of 4's in each unit; the number of 8's. *Ex.:* There is $\frac{1}{2}$ of 8 in the 4; there are $1\frac{1}{2}$ 8's in 12; 2 8's in 16 (showing the 2 8's), etc.

11. Show the 12's in each; the 16's; the 20's; the 24's.

12. What two equal units in each unit? What is $\frac{1}{2}$ of each unit?

13. 4 is an exact measure of how many of the units? 2 is an exact measure of how many of the units? of which units is 8 an exact measure?

For fixing the previous perceptions, use solids, surfaces and lines having the ratios of the units 4, 8, 12, 16, 20 and 24. Advance into the perception of new relations is no reason for ceasing to make use of those previously observed. Through new combinations the old become more and more significant, more truly known.

1. What is the ratio of 1 to 4? of 2 to 8? of 3 to 12? of 4 to 16? of 5 to 20? of 6 to 24?

2. What is true of the ratios of 1 to 4 and 2 to 8?

3. Name other ratios that are equal.

4. The ratio of 24 to 6 equals the ratio of what to 4?

5. 4 is the ratio of what to 5? of what to 6? of what to 2? of what to 3? of what to 4?

6. The ratio of 6 to 2 equals the ratio of what to 4?

7. Make statements similar to the following: The ratio of 2 to 4 equals the ratio of 6 to 12.

8. Make sentences like this: If the unit 24 represents $x¢$, the unit 12 represents $\frac{x¢}{2}$ (read, $\frac{1}{2}$ of $x¢$).

9. Like this: If $50¢$ is the cost of 4 doz. eggs, $3 \cdot 50¢$ is the cost of 12 doz. eggs.

10. Like this: If x oranges can be bought for $16¢$, $\frac{3 \cdot x}{2}$ oranges can be bought for $24¢$.

11. Like this: If the unit 24 costs $x¢$, the sum of the units 6 and 10 costs $\frac{2 \cdot x¢}{3}$ (read, $\frac{2}{3}$ of $x¢$).

12. Make sentences like this: If $75¢$ is the cost of 8, $\frac{3 \cdot 75¢}{2}$ is the cost of 12.

13. Like this: If $49¢$ is the cost of 20 lb. of salt, $\frac{3 \cdot 49¢}{4}$ is the cost of 15 lb.

14. Review. Change the sentences into questions.

Ex.: If $50¢$ is the cost of 4 doz. eggs, $3 \cdot 50¢$ is the cost of what?

Use the method of teaching the ratios of the units made of different numbers of 4's to teach the same ratios of units of 5's, 6's, 7's, 8's, '9s, and 10's, respectively.

a					
26	24	22	20	18	28
13	12	11	10	9	14

Ratios. — 1. What is the ratio of the smaller unit under a to the larger unit? of the larger to the smaller?

2. Tell which of the above units have the ratios $\frac{1}{2}$ and 2.

3. Make sentences like these: In 26 there are $2 \cdot 13$'s. $\frac{1}{2}$ is the relation of 13 to 26. 2 is the relation of 26 to 13. $2 \cdot 13\text{'s} = 26$. $\frac{2}{2}\frac{6}{} = 13$ (read, $\frac{1}{2}$ of $26 = 13$).

4. Make sentences like these: If \$8 is the cost of 26, $\$\frac{8}{2}$ is the cost of 13. If $x¢$ is the cost of 18, $\frac{x¢}{2}$ is the cost of 9. If 12 costs $17¢$, 24 costs $2 \cdot 17¢$.

1. At \$14 a month, what is the cost of 2 mo. board?

2. I owe a man \$28 and have only \$14. How much more do I need to pay the debt?

3. Mr. C spends 14¢ a week for papers. In what time does he spend 28¢ for them?

4. At 10¢ a dozen, a lady receives how much for 24 eggs?

5. Name as many units as you can which have the ratio 2.

Problems. — 1. There are 26 wk. in $\frac{1}{2}$ yr. 13 wk. equal what part of $\frac{1}{2}$ yr.? How many 13 wk. in 26 wk.?

2. If 12 cu. in. cost a dime, what is the cost of 24 cu. in.?

3. 22 tons of coal cost $100. What part of $100 do 11 tons cost?

4. 10 and 10 equal what? 10 doz. equal what part of 20 doz.?

5. 18 yd. of ribbon cost how many times as much as 9 yd.? What must be added to 9 yd. to make 18 yd.?

6. 28 is double what? 28 can be separated into how many 14's?

7. 14 dy. equal 2 wk. 28 dy. equal how many 2 wk.?

8. 2 is the relation of what to 14?

9. $\frac{1}{2}$ is the relation of what to 26?

10. The relation of 22 to 11 equals the relation of what to 4?

Imaging. — Ask questions like this: 13 and 13 equal what? Write answers of pupils on the blackboard. See

page 124, Teachers' Book. Have pupils observe, image and write the equations until they can do so readily.

13	11	12	9	10	14
13	11	12	9	10	14
26	22	24	18	20	28

What two equal units in 26? in 24? in 22? in 20? in 18?

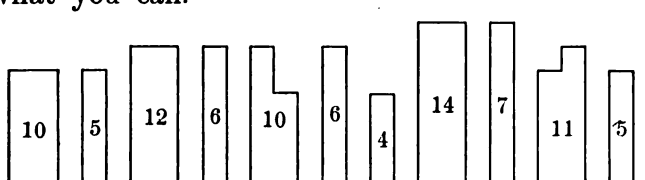
What is $\frac{2}{2}^{8}$ (read, $\frac{1}{2}$ of 28)? $\frac{2}{2}^{6}$? $\frac{2}{2}^{4}$? $\frac{2}{2}^{2}$? $\frac{2}{2}^{0}$? $\frac{1}{2}^{8}$?

14 13 12 11 10 9

Image two of each with sum. *Ex.:* $\begin{matrix}14\\14\\\hline 28\end{matrix}$; practice.

This is to be a mental picture not a visible representation.

Equations. — 1. Observe these units and tell what you can.



2. Make sentences like these: 10 is 4 more than 6; 5 is 6 less than 11.

3. 10 is how much more than 5? 5 and 5 equal what?

4. 12 is how much greater than 6? What must be added to 6 to make 12?

5. Show the 4 in the 10. How much more than the 4 do you see in the 10? 10 is how much more than 6? 6 and 4 equal what?

6. The sum of 7 and 7 is as large as what unit? 7 is how much less than 14?

7. 5 and what equal 11?

8. You can separate the unit 10 into 6 and what? the unit 11 into 5 and what? the unit 12 into 6 and what?

9. How much larger is 14 than 7? 11 than 5? 12 than 6? 10 than 4? 10 than 5?

Problems. — 1. Stella picked 4 qt. of cherries and 10 qt. of currants. How many more quarts of currants did she pick than of cherries? How many less of cherries than of currants? At $x¢$ a quart, what is the ratio of the money received for the cherries to the money received for the currants?

2. If a 11-ft. rope is needed, a 6-ft. rope is how much too short? If the 11-ft. rope costs $y¢$, the cost of the 6-ft. rope equals how many elevenths of $y¢$?

3. Matting which cost $6¢$ a yard was sold for $6¢$ a yd. more than the cost. For how much was it sold? The cost equaled what part of the selling price? The selling price less what equaled the cost?

If incorrect answers are given, *seek the cause.* It may be inattention due to fatigue, bad air, or method of presentation; or it may be inability to represent and hold in consciousness the terms to be compared — the cost and the selling price.

Whatever the cause, avoid anything like the following: "Well, Mary, what is the cost? What is the selling price? And 6¢ equals what part of 12¢? Then, if the cost is 6¢ and the selling price is 12¢, and 6¢ equals ½ of 12¢, the cost equals what part of the selling price?"

What better way could be devised for inducing idleness and vacancy of mind, for weakening the pupil and interfering with his grasp of the thought as a whole?[1]

Ascertain the mental condition of the pupil through what he does. We deceive ourselves when we put into his mouth the words he shall say and then take what he says as evidence of what he sees.

Weakness is the sure result of inactivity, whether the inactivity be due to work into which the learner *cannot* enter, or to methods which induce dawdling and mental dissipation.

4. Edith can hem a towel in 14 min. When she has worked 7 min., how many more minutes must she work? What part of the work does she do in 7 min.?

5. A lady paid 5¢ for a bunch of celery and 10¢ for a quart of peas. How much more did the peas cost than the celery?

6. Two boys started from the same place; one walked 5 yd. north and the other 5 yd. south. How far apart were they?

[1] See remarks on questioning, p. v, Preface, Teachers' Book.

Imaging — Method of Work. — Ask the following questions. Write answers of pupils on the blackboard, so that they may associate their thought with its expression.

5 and 5 equal what?
6 and 6 equal what?
6 and 4 equal what?
7 and 7 equal what?
5 and 6 equal what?

Answers.

a	*b*	*c*	*d*	*e*
5	6	6	7	5
5	6	4	7	6
10	12	10	14	11

Look at $\begin{array}{r} 5 \\ 5 \\ \hline 10 \end{array}$; write; practice.

Observe $\begin{array}{r} 6 \\ 6 \\ \hline 12 \end{array}$; image; write.

Observe $\begin{array}{rr} 5 & 6 \\ 5 & 6 \\ \hline 10 & 12 \end{array}$; image; write.

Observe $\begin{array}{r} 6 \\ 4 \\ \hline 10 \end{array}$; image; write; practice.

Observe $\begin{array}{rrr} 5 & 6 & 4 \\ 5 & 6 & 4 \\ \hline 10 & 12 & 10 \end{array}$; image; write; practice.

Continue adding one combination at a time until you can image and write the five readily.

Tests of Imaging. — Tell the combination under each letter, thus:

$\begin{array}{r} 5 \\ 5 \\ \hline 10 \end{array}$ is under *a*.

What combination is under *c*? *d*? *e*? *b*?

Image and think each combination with its sum, beginning at the top.[1]

[1] "Addition, as De Morgan insisted, is far more swiftly done by the eye alone; the tendency to use mental words should be withstood." — Francis Galton.

Write from memory the second combination from the left; the third from the right.

Write from memory the first figure in each combination, beginning at *a*: 5, 6, 6, etc.

Write the second figure in each, beginning at *a*.

Continue to work with these five combinations until they are indelibly fixed.

Write on blackboard:

5	6	6	7	5
5	6	4	7	6

Pronounce name of combination.

Observe and think 11, 14, etc.

Name sums from right to left, without observing the board; from left to right.

What is the second sum from the right? the third from the left?

Make columns of the combinations, omitting sums, thus:

a	*b*	*c*	*d*
5	5	7	6
5	6	7	6
6	7	6	4
5	7	6	6
7	6	6	7
7	6	4	7
6	4	5	5
6	6	5	6
4	5	6	5
6	5	5	5

Look at each column carefully, and image the sum of each combination of two figures.

Image, slowly at first, the combinations under *d*: 10, 11, 14, 10, 12. Practice.

Image, beginning at the top.

Image from right to left, thus: 12, 14, 11, 10.

Practice copying until you can reproduce the following five combinations and the sums from memory without any hesitation.

50	60	60	70	50
50	60	40	70	60
100				

Image the equations.

Think the sums of the tens only: 10, 12, etc.

1. **Problems.** — Draw a 10-in. line and a 6-in. line. How much must you add to the 6-in. line that its length may equal that of the 10-in. line? What is the difference in the length of the lines?

2. Mabel has 10¢ and Ruth has 4¢. How much must be added to Ruth's money that the sum may equal Mabel's? What, then, is the difference between 10¢ and 4¢?

3. 3 ft. must be added to a line that it may equal the height of a door. What is the difference between the length of the line and the height of the door?

4. If you buy a book worth 15¢ and give the seller 25¢, what does he give you in exchange for your 25¢? To find the difference between the 25¢ and the cost of the book, he adds how much to the cost of the book?

5. If you buy a 35¢ dinner and give the clerk 50¢, what do you receive for the 50¢? To find

the difference between the 50¢ and the price of the dinner, the clerk adds to what?

6. What must be added to 5 to make 11? 11 is how much more than 5? 5 is how much less than 11? The sum of 5 and what equals 11?

7. If you add to 40¢ until the sum equals $1, you have found the difference between what?

8. The sum of 9¢ and what equals 25¢? What, then, is the difference between 9¢ and 25¢?

9. If to a you must add b that the sum may equal c, what is the difference between c and a?

Make problems similar to 9. Represent by drawing.

1. At 5¢ each, what is the cost of 2 tops?

2. At $50 each, what is the cost of 2 carriages?

3. A 12-yd. line is how much longer than a 6-yd. line?

4. A 120-yd. fence is how much longer than a 60-yd. fence?

5. 60 is how much less than 120?

6. A 4-ft. window shade is how much too short for a window 10 ft. in height?

7. In a 40-acre field there is how much less land than in a 100-acre field? 40 and what equal 100? 100 less what equals 40? 100 less what equals 60? 60 and 40 equals how many?

8. What must be added to 40¢ that the sum may equal 100¢? to 60¢ that the sum may equal 100¢?

9. Fred has $1 in his bank and Harry has 60¢ in his. How much must be added to Harry's money that it may equal Fred's?

10. What is the difference between the sum in Fred's bank and the sum in Harry's?

11. 14 dy. are how many more than 7 dy.? 2 wk. are how much more than 7 dy.? 7 dy. and how many days are 14 dy.?

12. Lulu had 11¢. She paid 5¢ for car fare. How much had she left?

13. If the money in my purse equals the sum of $5 and $6, how much money in the purse?

14. A merchant received $11 for a hat and coat. If the price of the hat was $5, what was the price of the coat?

15. 11¢ equal the sum of Mary's and Kate's money. Mary has 5¢; how much has Kate? The sum of their money is how much more than Mary's money? how much more than Kate's?

Equations — Imaging. — Ask the following questions and write the answers on the blackboard:

	Answers.				
5 and what equal 10?					
6 and what equal 12?	10	12	10	14	11
6 and what equal 10?	5	6	6	7	5
7 and what equal 14?	5	6	4	7	6
5 and what equal 11?					

Observe $\frac{10}{5}$; image; write. Observe $\frac{12}{6}$; image; write; practice.

Image $\frac{11}{5}$ $\frac{12}{6}$; increase the amount to be remembered until you can write the five combinations from memory.

Image and tell where the different combinations are. *Ex.:* 6, 4, 10 is the third from the right.

Write the combinations without the differences.

Practice writing and erasing differences until you can write them instantly.

Write table on blackboard without differences. Have pupils practice imaging differences. In imaging, associate the three symbols in each equation.

Practice pronouncing differences from right to left and from left to right.

Master this table before beginning another. Work with the second as with the first.

Review first and second before beginning the third.

Work with the following table in the same manner as with the similar table on page 69.

38	36	32	34	30
19	18	16	17	15

Ratios. — 1. \$7 is the cost of 19. What is the cost of 38?

2. The sum of 38 and 19 equals how many 19's?

3. 38 weighs 35 lb. What part of 35 equals the weight of 19?

4. In 36 there are 3 doz. In 18 there is what part of 3 doz.?

5. A shadow cast by a tree 36 ft. high is how many times as long as a shadow cast by a tree 18 ft. high?

6. What must be added to 17 to make 34?

7. There are 2·8's in 16. How many 2·8's in 32? Why?

8. There are 6·5's in 30. What part of 6·5's are there in 15?

9. Make sentences like these: 2 is the relation of 38 to 19. $\frac{1}{2}$ is the relation of 18 to 36.

10. 2 is the relation of what unit to 17?

11. Name units that have the relation 2.

Equations — Imaging. — Ask questions like this: **15** and **15** equal what?

Write answers on the blackboard, thus: $\begin{array}{r} 15 \\ 15. \\ \hline 30 \end{array}$

Pupils practice observing and imaging the units until they can do so readily.

19	18	17	16	15
19	18	17	16	15
38	36	34	32	30

What two equal units in 38? in 36? in 34? in 32? in 30?

What is $\frac{38}{2}$? $\frac{36}{2}$? $\frac{34}{2}$? $\frac{32}{2}$? $\frac{30}{2}$?

19 18 17 16 15

Image two of each with sum. *Ex.:* $\frac{15}{30}$; practice.

48	46	44	42	40
24	23	22	21	20

Ratios. — 1. 48 buttons equal 4 doz. 24 buttons equal how many doz.?

2. There are 3 · 8's in 24. How many 3 · 8's in 48?

3. 23 lb. last 7 wk. What time do 46 lb. last?

4. 22 equals 2 · 11's. 44 equals how many 11's?

5. 21 dy. equal 3 wk. 42 dy. equal how many weeks?

6. 20¢ equals 4 nickels. 40¢ equals how many?

7. 2 is the relation of what to 21 dy.?

8. $\frac{1}{2}$ is the relation of what to 42 dy.?

9. Name units that have the ratio 2.

Ex.: 2 is the ratio of 44 to 22; of 6 to 3; of $\frac{1}{2}$ to $\frac{1}{4}$.

10. Make lists of the units that have the ratio 2.

Equations — Imaging. — Ask questions like this: 24 and 24 equal what? Write answers on the blackboard, thus: $\frac{24}{24.}$
$\overline{48}$

Pupils practice observing and imaging the units until they can do so readily.

24	23	22	21	20
24	23	22	21	20
48	46	44	42	40

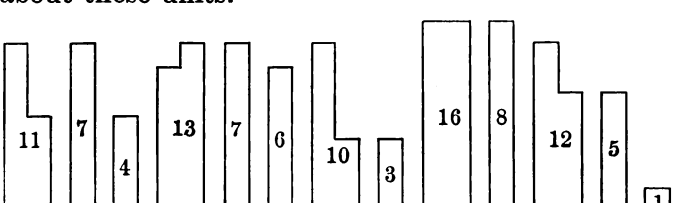
What two equal units in 48? in 46? in 44? in 42? in 40?

What is $\frac{48}{2}$? $\frac{46}{2}$? $\frac{44}{2}$? $\frac{40}{2}$? $\frac{42}{2}$?

24 23 22 21 20

Image two of each with sum. *Ex.:* $\begin{matrix}20\\20\\\hline40\end{matrix}$; practice.

Equations. — 1. Observe and tell what you can about these units.



2. Make sentences like this: 11 can be separated into 7 and 4.

3. Make sentences like this: 11 is 4 larger than 7.

4. Make sentences like this: 4 is 7 less than 11.

5. What two units do you see in 11? in 13? in 16? in 12?

6. What must be added to 4 to make 11? to 7 to make 13? to 3 to make 10? to 8 to make 16? to 5 to make 12?

7. 7 and what equal 11? 6 and what equal 13? 7 and what equal 10? 8 and what equal 16?

8. 11 is as large as 7 and what? 13 is as large as 6 and what? 10 is as large as 7 and what? 16 is as large as 8 and what? 12 is as large as 5 and what?

9. 7 and 4 equal what? 7 and 6 equal what? 3 and 7 equal what? 8 and 8 equal what? 7 and 5 equal what?

Problems. — 1. A grocer bought cherries at 7¢ a quart and sold them for 12¢ a quart. How much did he gain on each quart?

2. A dealer sold pins at 12¢ a paper, which was 5¢ a paper more than they cost him. How much did they cost him?

3. 12 is how much more than 5? than 7? 5 is how much less than 12? 5 and what equal 12?

4. Walter wishes to buy a glass of lemonade, which costs 10¢, but he has only 7¢. How much more does he need? 3¢ equals what part of 10¢? 3¢ equals what part of the sum of 7¢ and 3¢?

5. A hall is 16 ft. long and 8 ft. wide. How many more feet in the length than in the width?

6. A ball cost 8¢ and a top 8¢. What was the cost of both?

7. William caught a ball 11 times and missed it 4 times. How many more times did he catch than miss?

8. Percy read 4 pages yesterday and 7 to-day. How much in all?

9. There are 13 crayons in a box and 6 on a table. How many more in the box than on the table?

10. Nettie is 13 yr. old and Hattie 7 yr. In how many years will Hattie be as old as Nettie now is?

11. At $8 a week, how much does a man earn in 2 wk.? At $6\frac{1}{2}$ a week, how much does he earn in 2 wk.?

12. A room is 11 ft. in length and a strip of carpet 7 ft. How much must be added to the length of the carpet that its length may equal that of the room? What is the difference between the length of the carpet and the length of the room? If you add to 7 ft. the difference between 7 ft. and 11 ft., what is the sum?

Equations. — Ask the following questions, and write answers on the blackboard. See method of work on page 74.

7 and 4 equal what?
6 and 7 equal what?
7 and 3 equal what?
8 and 8 equal what?
5 and 7 equal what?

Answers.

7	6	7	8	5
4	7	3	8	7
11	13	10	16	12

a	b	c	d
7	6	3	6
4	7	7	7
6	7	7	4
7	4	6	7
7	3	7	5
3	7	5	7
8	8	8	7
8	5	8	3
5	8	4	8
7	8	7	8

Ask the following questions, write answers on the blackboard, and use by methods suggested on page 79.

4 and what equal 11?
7 and what equal 13?
3 and what equal 10?
8 and what equal 16?
7 and what equal 12?

Answers.

11	13	10	16	12
4	7	3	8	7
7	6	7	8	5

7 and what equal 11?
6 and what equal 13?
7 and what equal 10?
8 and what equal 16?
5 and what equal 12?

Answers.

11	13	10	16	12
7	6	7	8	5
4	7	3	8	7

60	58	56	54	52	50
30	29	28	27	26	25

Ratios. — See method of work, page 69.

1. 60 equals 4 · 15's. 30 equals what part of 4 · 15's?

2. 30 equals $2\frac{1}{2}$ doz. 60 equals how many dozen?

3. 29 weighs 72 oz. How many 72 oz. in the weight of 58?

4. 29 and 58 equal how many 29's?

5. There are 8 wk. in 56 dy. There is what part of 8 wk. in 28 dy.?

6. 27 ft. equal 9 yd. 54 ft. equal how many yards?

7. 26 equals 2 · 13's. 52 equals how many 13's?

8. 52 wk. equal 1 yr.; 26 wk. equal what part of a year?

9. The number of dimes in 50¢ is how many times as great as the number in 25¢?

10. The paper you can buy for 25¢ equals what part of the paper you can buy for 50¢?

11. A 54-ft. line is how many times as long as a 27-ft. line?

12. In room *a* there are 25 pupils, which equals $\frac{1}{2}$ the number in room *b*. What is the number in room *b*?

13. A florist received \$15 for $\frac{1}{4}$ of his flowers. At this rate what would he receive for $\frac{1}{2}$ of them? for $\frac{4}{4}$ of them?

14. What is the ratio of 29 to 58?

15. At \$1 a day, how much does a man earn in 29 dy.? in twice 29 dy.?

Name units that have the ratio 2.

Write ten pairs of units that have the ratio $\frac{1}{2}$.

Ex.: 29 58
3 6
$\frac{1}{4}$ $\frac{1}{2}$

Imaging. — Ask questions like this: 30 and 30 equal what? Write answers on the blackboard, thus: $\begin{array}{r} 30 \\ 30. \\ \hline 60 \end{array}$

Pupils practice observing and imaging until they can do so readily.

$$\begin{array}{cccccc} 30 & 29 & 28 & 27 & 26 & 25 \\ 30 & 29 & 28 & 27 & 26 & 25 \\ \hline 60 & 58 & 56 & 54 & 52 & 50 \end{array}$$

What two equal units in 60? in 58? in 56? in 54? in 52? in 50?

What is $\frac{60}{2}$? $\frac{58}{2}$? $\frac{56}{2}$? $\frac{54}{2}$? $\frac{52}{2}$? $\frac{50}{2}$?

30 29 28 27 26 25

Image two of each with sum. *Ex.:* $\begin{array}{r} 25 \\ 25. \\ \hline 50 \end{array}$

30	27	24	21	18
20	18	16	14	12
10	9	8	7	6

Ratios. — 1. Which units have the ratios 2 and $\frac{1}{2}$? Which have the ratios 3 and $\frac{1}{3}$? $\frac{2}{3}$ and $\frac{3}{2}$?

2. Draw units having the ratio of 27 to 18; of 27 to 9; of 16 to 24; of 8 to 24; of 7 to 14; of 7 to 21.

3. The ratio 27 to 18 equals the ratio of what to 16? of what to 14? of what to 12?

4. In each set compare each unit with the other two, thus:

$10 = \frac{20}{2}, \frac{30}{3}$.
$20 = 2$ times 10, $\frac{2 \cdot 30}{3}$.
$30 = 3$ times 10, $\frac{3 \cdot 20}{2}$.

5. Practice imaging the units and thinking the comparisons.

6. Practice imaging the units and writing the comparisons.

7. Make sentences like these: In 18 there are 3 · 6's; in 12, 2. 3 is the ratio of 18 to 6; of 24 to 8; etc. $\frac{3}{2}$ is the ratio of 18 to 12; of 27 to 18; etc. $\frac{1}{3}$ is the ratio of 6 to 18; of 8 to 24; etc. $\frac{2}{3}$ is the ratio of 12 to 18; of 20 to 30; etc.

8. Make sentences like this: If 27 costs \$15, 18 costs $\frac{2 \cdot \$15}{3}$ and 9, $\frac{\$15}{3}$.

9. Make sentences like this: If 14 weighs 17 lb., 7 weighs $\frac{1}{2}$ lb. and 21, $\frac{3}{2}$ lb.

1. If 10¢ buys 2, 20¢ buys how many? 30¢?
2. 18 ft. equal 6 yd. 9 ft. equal what? 27 ft. equal how many 3 yd.?
3. 16 oz. equal 1 lb. 8 oz. equal what? 24 oz. equal what?
4. 16 equals 4 4's. 8 equals how many 4's? 24 equals how many 4's?
5. If 14 costs \$10, what is the cost of 7? of 21?
6. 12 equals 1 doz. 6 equals what? 18 equals what?
7. Make sentences like this: 3 is the ratio of 27 to 9.
8. Make sentences like this: $\frac{2}{3}$ is the ratio of 16 to 24.
9. Make sentences like this: $\frac{3}{2}$ is the ratio of 18 to 12.

Things and words become significant through the mind's relating. The presentation should cause the use of the terms 9, 18, 27, etc., to be the result of an act of judging. But we do not foster activity in judging by withholding the language needed for thinking.[1]

10. $\frac{3}{2}$ is the ratio of what to 16? to 20? to 14?

[1] "There is no such thing as a valuable observation and investigation of natural objects without language in which to embody the results at every step." — Geo. P. Brown.

11. Write names of different sets of units that have the ratios found in the table.

Ex.: 9, 18, 27
10, 20, 30
2 ft., 4 ft., 6 ft.
$\frac{1}{3}$, $\frac{2}{3}$, 1

What three equal units in 30? in 27? in 24? in 21? in 18?

What is $\frac{30}{3}$ (read, $\frac{1}{3}$ of 30)? $\frac{27}{3}$? $\frac{24}{3}$? $\frac{21}{3}$? $\frac{18}{3}$?

Practice reading at sight 10, etc.

$\frac{30}{3}$ $\frac{27}{3}$ $\frac{24}{3}$ $\frac{21}{3}$ $\frac{18}{3}$

Image three of each with the sum. *Ex.:* $\begin{array}{r} 6 \\ 6 \\ \hline 18 \end{array}$; practice.

10 9 8 7 6

Problems. — 1. If 30¢ buys 1 doz. peaches, 20¢ buys what part of 1 doz.?

2. Fred spent 21¢ for note books, at 7¢ each. How many did he buy?

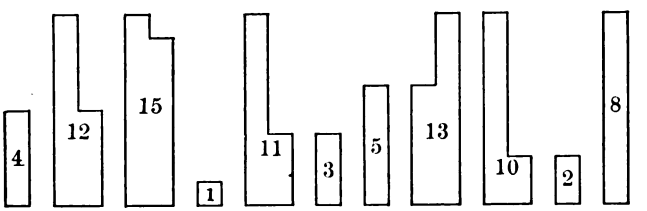
3. A piece of twine is 18 in. long and a piece of tape 12 in. What is the ratio of the length of the twine to the length of the tape? Into how many 6-in. pieces could you cut the twine?

4. If $\frac{1}{2}$ lb. of butter costs 9¢, what does 1 lb., or 16 oz., cost? what is the cost of $1\frac{1}{2}$ lb? $1\frac{1}{2}$ lb. costs how many times as much as $\frac{1}{2}$ lb.?

5. A man paid $27 for a cow and sold it for $\frac{2}{3}$ as much as he paid. For how much did he sell it? How much did he lose? The loss equals what part of $27?

6. At 20¢ a yard, what will $1\frac{1}{2}$ yd. of ribbon cost? at 16¢ a yd.?

7. At 14¢ a quart, $\frac{1}{2}$ qt. of beans costs how much?



Equations. — 1. Observe units and tell what you see.

2. Make sentences like this: 12 can be separated into 8 and 4.

3. Make sentences like this: 12 is 8 larger than 4.

4. Make sentences like this: 4 is 8 less than 12.

5. What two units do you see in 12? in 15? in 11? in 13? in 10?

6. 4 and 8 equal what? 8 and 7 equal what? 8 and 3 equal what? 8 and 5 equal what? 2 and 6 equal what?

7. 4 and what equal 12? 8 and what equal 15? 5 and what equal 13? 2 and what equal 10?

Ask the following questions, write answers on the blackboard, and use the methods suggested before:

	Answers.				
4 and 8 equal what?					
8 and 7 equal what?	8	7	8	5	8
3 and 8 equal what?	4	8	3	8	2
8 and 5 equal what?	12	15	11	13	10
2 and 8 equal what?					

a	*b*	*c*	*d*
8	7	2	8
4	8	8	5
7	8	8	8
8	4	7	4
8	8	5	7
3	5	8	8
5	3	4	8
8	8	8	3
8	4	3	8
2	8	8	2

Ask the following questions, write answers on the blackboard, and drill as suggested on page 79.

	Answers.				
8 and what equal 12?					
7 and what equal 15?	12	15	11	13	10
8 and what equal 11?	8	7	8	8	8
8 and what equal 13?	4	8	3	5	2
8 and what equal 10?					

	Answers.				
4 and what equal 12?	12	15	11	13	10
8 and what equal 15?	4	8	3	5	2
3 and what equal 11?	8	7	8	8	8
5 and what equal 13?					
2 and what equal 10?					

1. Draw a rectangle containing 12 sq. in. Draw one containing 8 sq. in. How many more square inches in the first than in the second? How many square inches must be added to the smaller rectangle to make it equal the first? From the rectangle containing 12 sq. in. erase 4. How much of the rectangle is left?

2. Carrie paid 4¢ for candy, which was $\frac{1}{2}$ as much as she paid for rolls and $\frac{1}{3}$ as much as for a pie. How much did she pay for the rolls and how much for the pie?

3. I gave a clerk a dime to pay for a bottle of ink which cost 8¢. What did I receive in change? 8¢ and what equal 10¢? 10¢ is how much more than 8¢?

4. How many 2¢ stamps can I buy for 8¢? for 10¢? $\frac{1}{4}$ of 8 equals what part of 10?

5. After 3 ft. were cut from an 11-ft. pole, what was the length of the pole?

6. Fred earned 8¢ and found 3¢. How much had he then?

7. A lady wishes to buy a lounge worth $13. She has $8. How much more does she need?

8. George lives 5 blocks west of the schoolhouse and Alfred 8 blocks east. How far apart do they live?

9. The width of Mr. B's lot is 15 yd. and of Mr. C's 8 yd. How much wider is Mr. B's lot than Mr. C's?

10. How much must be added to the width of Mr. C's to make it equal Mr. B's?

11. Draw a line 5 yd. long. Erase 7 ft. How long is the remaining line?

The basis of mathematical reasoning is the perception of the relative magnitude of things. The *expression* of definite relations cannot do duty for the *perception* of such relations. Nothing but repeated acts of observing and comparing will bring the relations of magnitude before the child as realities.

Ratios. — Work with this table as with similar preceding tables.[1]

45	42	39	36	33
30	28	26	24	22
15	14	13	12	11

1. If you give 6 five-cent pieces for 30¢, what part of 6 five-cent pieces ought you to give for 15¢? for 45¢?

[1] "The study of mathematics is unceasingly calling forth the faculties of observation and comparison. . . I might go on piling instance upon instance to show the paramount importance of the faculty of observation to the process of mathematical discovery." — Professor Sylvester.

2. 42 dy. equal 6 wk. 28 dy. equal how many weeks?

3. 28 equals what part of 42? 4 equals what part of 6?

4. 26 wk. equal $\frac{1}{2}$ yr. What do 13 wk. equal? 39 wk.?

5. 12 in. equal 1 ft. What do 24 in. equal? 36 in.?

6. 11 costs 19. 22 costs how many 19's? 33 costs what?

7. Make sentences like this: 3 is the ratio of 42 to 14.

8. Make sentences like this: $\frac{3}{2}$ is the ratio of 39 to 26.

9. Make sentences like this: The ratio of 45 to 15 equals the ratio of 33 to 11.

10. 3 is the ratio of what to 15? 3 is the ratio of what to 11?

What 3 equal units in 45? in 42? in 39? in 36? in 33?

What is $\frac{45}{3}$ (read, $\frac{1}{3}$ of 45)? $\frac{42}{3}$? $\frac{39}{3}$? $\frac{36}{3}$? $\frac{33}{3}$?

Practice reading at sight 15, etc.

$\frac{45}{3}$ $\frac{42}{3}$ $\frac{39}{3}$ $\frac{36}{3}$ $\frac{33}{3}$

Practice reading at sight 30, etc.

$\frac{2\cdot45}{3}=?$ $\frac{2\cdot42}{3}=?$ $\frac{2\cdot39}{3}=?$ $\frac{2\cdot36}{3}=?$ $\frac{2\cdot33}{3}=?$

Image three of each with the sum.

15 14 13 12 11

Practice.

Problems. — 1. At 13¢ a box, what will 2 boxes of cherries cost? 3 boxes?

2. At 26¢ a pound, what will $1\frac{1}{2}$ lb. of butter cost?

3. If 26 yr. is $\frac{2}{3}$ of Mr. B's age, his age equals how many halves of 26 yr.?

4. If $\frac{1}{4}$ lb. of candy costs 15¢, what will $\frac{1}{2}$ lb. cost?

5. What is the ratio of 45 to 15? If you can buy $\frac{1}{4}$ lb. of candy for 15¢, what part of 1 lb. can you buy for 45¢?

6. What is the ratio of 14 to 28? of 42 to 14? of 42 to $\frac{1}{2}$ of 28? of 42 to 28?

7. 28 men can do how many times as much work in a day as 14 men? 42 men can do how many times as much as 14 men? What is the ratio of the work 42 men do to the work 28 men do in the same time?

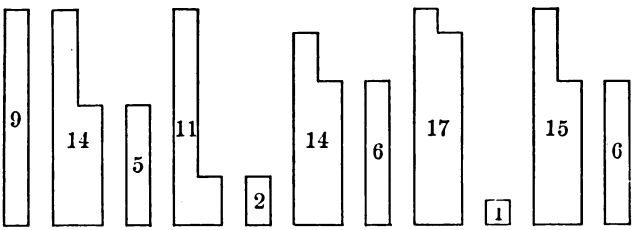
8. Draw lines to represent the work done by 14, by 28, and by 42 men in a day?

9. What part of a day would it take the 28 men to do the smallest piece of work? the 42 men? How many days would it take the 14 men to do what the 28 men can do in a day? How long to do what the 42 men can do in a day?

10. Laura and Jean each paid $\frac{2}{3}$ as much for a story book as for a doll. Laura paid 24¢ for her book. What did she pay for her doll? Jean paid 22¢ for her book. What did she pay for her doll?

11. At 22¢ a yard, what is the cost of $\frac{1}{2}$ yd. of lawn? $1\frac{1}{2}$ yd.?

Equations. — 1. Observe units and tell what you see.



2. 14 is 9 larger than what? 11 is 9 larger than what?

3. 14 is 8 larger than what? 17 can be separated into 9 and what? 15 into 9 and what?

4. 9 and 5 equal what? 2 and 9 equal what? 8 and 6 equal what? 9 and 8 equal what? 5 and 9 equal what?

5. 5 and what equal 14? 2 and what equal 11? 6 and what equal 14? 9 and what equal 17? 6 and what equal 15?

1. In a class there are 14 boys and 6 girls. How many more boys than girls?

2. There are 14 cows in one meadow and 5 in another. How many more in one field than in the other?

3. Carrie has 14 yd. of ruffling to hem. When she has hemmed 9 yd., how much more will she have to hem?

4. A room is 17 ft. long and 9 ft. wide. The length is how much greater than the width? The width is how much less than the length?

5. Make a dot on the blackboard. From the dot draw a line to the right 8 in. long. Draw one to the left 6 in. long. What is the length of the entire line?

6. A lane is 11 rd. from north to south. A tree 2 rd. from the north entrance is how far from the south entrance?

See method of work, page 79.

	Answers.				
9 and what equal 14?					
9 and what equal 11?	14	11	14	17	15
8 and what equal 14?	9	9	8	9	9
9 and what equal 17?	5	2	6	8	6
9 and what equal 15?					

	Answers.				
5 and what equal 14?					
2 and what equal 11?	14	11	14	17	15
6 and what equal 14?	5	2	6	8	6
8 and what equal 17?	9	9	8	9	9
6 and what equal 15?					

See method of work, page 74.

	Answers.				
9 and 5 equal what?					
2 and 9 equal what?	9	9	8	9	9
6 and 8 equal what?	5	2	6	8	6
8 and 9 equal what?	14	11	14	17	15
6 and 9 equal what?					

a	b	c	d
9	2	8	8
5	9	6	9
9	5	9	9
2	9	2	5
8	9	5	2
6	8	9	9
9	6	6	6
8	8	9	8
9	6	8	9
6	9	6	6

40	36	32	28	24
30	27	24	21	18
20	18	16	14	12
10	9	8	7	6

Ratios. — See method of work, page 88.

1. Make sentences like these: In 24 there are 4 · 6's; in 18, 3; in 12, 2. 4 is the ratio of 24 to 6; 3, of 18 to 6; 2, of 12 to 6. $\frac{1}{2}$ is the ratio of 6 to 12; $\frac{1}{3}$, of 6 to 18; $\frac{1}{4}$, of 6 to 24. 2 is the ratio of 12 to 6; $\frac{2}{3}$, of 12 to 18; $\frac{1}{2}$, of 12 to 24.

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3 is the ratio of 18 to 6; $\frac{3}{2}$, of 18 to 12; $\frac{3}{4}$, of 18 to 24. 4 is the ratio of 24 to 6; 2, of 24 to 12; $\frac{4}{3}$, of 24 to 18.

2. Make sentences like this: If 24 lasts 5 yr., 8 lasts $\frac{5}{3}$ yr.; 16, $\frac{2 \cdot 5}{3}$ yr.; 32, $\frac{4 \cdot 5}{3}$ yr.

3. What is the ratio of the cost of 18 ft. of wire to the cost of 12 ft.?

4. If 3 bbl. of flour can be bought for \$18, how many bbl. can be bought for \$24?

5. If 24 bananas cost 20¢, what is the cost of 32 bananas?

Let pupils make problems involving above ratios.

1. There are 2 · 5's in 10. How many 5's in 20? in 30? in 40?

2. There are 3 · 3's in 9. How many 3's in 18? in 27? in 36?

3. There are 2 · 4's in 8. How many 4's in 16? in 24? in 32?

4. There are 4 · 7's in 28. In 21 there is what part of 4 · 7's? in 14? in 7?

5. There are 2 · 3's in 6. How many 2 · 3's in 12? in 18? in 24?

What four equal units in 40? in 36? in 32? in 28? in 24?

What is $\frac{4 \cdot 0}{2}$? of 36? of 32? of 28? of 24?

Practice reading the following at sight 20, 18, etc.

$$\frac{4 \cdot 0}{2} \qquad \frac{3 \cdot 6}{2} \qquad \frac{3 \cdot 2}{2} \qquad \frac{2 \cdot 8}{2} \qquad \frac{2 \cdot 4}{2}$$

Practice reading at sight 10, 9, etc.

$\frac{40}{4}$ $\frac{36}{4}$ $\frac{32}{4}$ $\frac{28}{4}$ $\frac{24}{4}$

Practice reading at sight 30, 27, etc.

$\frac{3\cdot40}{4}$ $\frac{3\cdot36}{4}$ $\frac{3\cdot32}{4}$ $\frac{3\cdot28}{4}$ $\frac{3\cdot24}{4}$

Practice reading at sight 40, 36, etc.

$\frac{4\cdot30}{3}$ $\frac{4\cdot27}{3}$ $\frac{4\cdot24}{3}$ $\frac{4\cdot21}{3}$ $\frac{4\cdot18}{3}$

$$\begin{array}{r} 6 \\ 6 \\ 6 \\ \underline{6} \\ 24 \end{array}$$

Image four of each with sum. *Ex.:* 6; practice.

10 9 8 7 6

Problems. — 1. 20 things equal 1 score. What do 10 equal? 30? 40?

2. If 20¢ equals 4 nickels, what does 10¢ equal? 30¢? 40¢?

3. If a is the number you can buy for 30¢, what part of a is the number you can buy for 10¢? for 20¢? What can you buy for 40¢?

4. If 18 sq. ft. equal 2 sq. yd., what do 9 sq. ft. equal? 27 sq. ft.? 36 sq. ft.?

5. 9 equals $\frac{3}{4}$ of a dozen. How many $\frac{3}{4}$ doz. in 18? in 27? in 36?

6. 8 sq. ft. equal how many squares 2 ft. long?

[1] Do not read $\frac{3}{4}$ of 40 nor $\frac{1}{4}$ of 3·40's, but at sight read *30*.

What number of 2-ft. squares do 16 sq. ft. equal? 24 sq. ft.? 32 sq. ft.?

7. How many weeks in 7 dy.? in 14 dy.? in 21 dy.? in 28 dy.?

8. If 5 of every 7 dy. are school days, how many school days in 14 dy.? in 21 dy.? in 28 dy.?

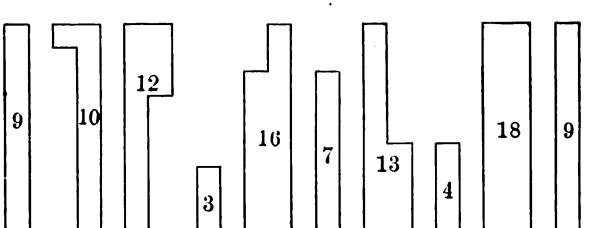
9. 6 in. equal what part of 1 ft.? 12 in. equal how many $\frac{1}{2}$ ft.? How many $\frac{1}{2}$ ft. in 18 in.? in 24 in.?

10. There are 24 sheets of paper in a quire. 12 sheets equal what part of a quire? 6 sheets? 18 sheets?

11. A score equals how many? A dozen equals how many? A square yard equals how many square feet? A quire equals how many sheets of paper?

12. Make sentences like this: $\frac{3}{2}$ is the ratio of 27 to 18.

13. $\frac{3}{2}$ is the ratio of what to 20? of what to 12?



Equations. — 1. Study diagram and tell what you see.

2. 10 can be separated into 9 and what? 12 into 9 and what? 16 into 7 and what? 18 into 9 and what?

3. 10 is how much larger than 1? 12 than 3? 16 than 7? 18 than 9?

4. 1 and what equal 10? 9 and what equal 12? 7 and what equal 16? 4 and what equal 13? 9 and what equal 13? 9 and what equal 18?

Problems.—1. Mr. Wilson dug 18 bu. of potatoes and his son 9 bu. How many more bushels did Mr. Wilson dig than his son?

2. A man wishes to go from M to N, which are 13 mi. apart. When he has traveled 4 mi., how much farther must he travel? What must be added to 9 mi. to make 13 mi.?

3. If a chain cost $16 and was sold for $9, what was the loss?

4. A man earns $12 a week and pays $3 for his board. What he earns is how much more than he pays for board?

5. There are 9 qt. of milk in one can and 3 qt. in another. How many in both?

6. A hat cost $7 and a cloak $9. What did both cost? If a lady has only money enough to pay for the hat, how much more does she need to pay for both the cloak and the hat?

What is the difference between the cost of both and the cost of the hat?

What is the difference between $7 and the sum of $9 and $7? If to $9 you add $7, what is the sum?

7. Frank has 13 marbles and Robert 9. How many must be added to Robert's that he may have as many as Frank? What is the difference between 9 and 13?

8. If the line a must be extended 7 in. that its length may be 16 in., what is the length of a? What is the difference between 16 in. and 9 in.?

See method of work, page 74.

	Answers.				
1 and 9 equal what?					
9 and 3 equal what?	9	3	7	9	9
9 and 7 equal what?	1	9	9	4	9
4 and 9 equal what?	10	12	16	13	18
9 and 9 equal what?					

a	b	c	d
9	9	7	9
1	3	9	9
3	9	9	9
9	7	3	9
7	4	1	9
9	9	9	4
9	1	9	7
4	9	4	9
9	9	7	9
9	9	9	1

See method of work, pages 78, 79.

	Answers.				
9 and what equal 10?					
3 and what equal 12?	10	12	16	13	18
7 and what equal 16?	9	3	7	9	9
4 and what equal 13?	1	9	9	4	9
9 and what equal 18?					

	Answers.				
1 and what equal 10?					
9 and what equal 12?	10	12	16	13	18
9 and what equal 16?	1	9	9	4	9
4 and what equal 13?	9	3	7	9	9
9 and what equal 18?					

Ratios. — See method of work, page 88.

60	56	52	48	44
45	42	39	36	33
30	28	26	24	22
15	14	13	12	11

1. 60 equals 5 doz. 30 equals what part of 5 doz.? 15 equals what part? 45 equals what part?

2. How many weeks in 14 dy.? in 42 dy.? in 28 dy.? in 56 dy.?

3. 26 wk. equal $\frac{1}{2}$ yr. 13 wk. equal what part of $\frac{1}{2}$ yr.? 39 wk. equal what part of a year?

4. How many dozen in 12? in 24? in 36? in 48?

5. 11 equals $2 \cdot 5\frac{1}{2}$. 22 equals how many $5\frac{1}{2}$? 33 equals how many? 44 equals how many?

6. If there are $3 \cdot 5$'s in 15, how many $3 \cdot 5$'s in 30? in 45? in 60?

7. 14 equals $2 \cdot 7$'s. 28 equals how many $2 \cdot 7$'s? 42 equals how many? 56 equals how many?

8. If 13 wk. equal $\frac{1}{4}$ yr., 26 wk. equal what? 39 wk. equal what?

9. 36 in. equal 1 yd. What do 12 in. equal? 24 in.? 48 in.?

10. 33 costs \$5. What part of \$5 equals the cost of 22? of 11? of 44?

11. There are $3 \cdot 5$'s in 15. How many in 30? in 45? in 60?

12. There are $2 \cdot 7$'s in 14. How many in 28? in 42? in 56?

13. There are $4 \cdot 13$'s in 52. What part of $4 \cdot 13$'s in 39? in 26?

14. There are $2 \cdot 6$'s in 12. How many in 24? in 36? in 48?

15. There are $4 \cdot 11$'s in 44? What part of $4 \cdot 11$'s in 33? in 22?

16. What four equal units in 60? in 52? in 56? in 48? in 44?

What is $\frac{60}{2}$? of 52? of 56? of 48? of 44?

Practice reading results at sight.

$\frac{52}{2}$ $\frac{60}{2}$ $\frac{56}{2}$ $\frac{44}{2}$ $\frac{48}{2}$

$\frac{56}{4}$ $\frac{60}{4}$ $\frac{48}{4}$ $\frac{44}{4}$ $\frac{52}{4}$

$\frac{3\cdot60}{4}$ $\frac{3\cdot48}{4}$ $\frac{3\cdot56}{4}$ $\frac{3\cdot52}{4}$ $\frac{3\cdot44}{4}$

$\frac{4\cdot45}{3}$ $\frac{4\cdot39}{3}$ $\frac{4\cdot42}{3}$ $\frac{4\cdot36}{3}$ $\frac{4\cdot33}{3}$

Image four of each of the following with the sum. Practice.

15 14 12 13 11

Problems. — 1. A 14-ft. line equals what part of a 56-ft. line?

2. At 56¢ a bushel, what will $\frac{1}{4}$ bu. of wheat cost?

3. What is the ratio of 14 to 42? of 56 to 14? of 56 to $\frac{1}{3}$ of 42?

56 equals how many $\frac{1}{3}$ of 42?

If 42 ft. of lumber cost $\$x$, 56 ft. will cost how many $\frac{1}{3}$ of $\$x$?

4. If 56 costs $\$a$, 42 will cost what part of $\$a$?

5. What is the ratio of 48 to 12? of 36 to 12? of 48 to 36?

6. The ratio of 9 to 3 equals the ratio of what to 16? of what to 20? of what to 12? of what to 5?

7. If $\frac{3}{4}$ of an acre is worth \$33, what is 1 acre worth? $\frac{1}{2}$ acre? $\frac{1}{4}$ acre?

8. In 60 min. we can do how many times as much as in 15 min.?

9. A car which travels 20 mi. an hour will travel how far in 45 min.?

10. What is the ratio of the amount a man earns in 33 dy. to the amount he earns in 22 dy.?

35	30	25	20	15
28	24	20	16	12
21	18	15	12	9
14	12	10	8	6
7	6	5	4	3

Ratios. — 1. If the largest unit in each column represents 1, what part of 1 is represented by each of the other units?

2. Tell all you can about the relation of the units in one of the sets.

3. In each set compare each unit with the other four, thus:

$7 = \frac{14}{2}, \frac{21}{3}, \frac{28}{4}, \frac{35}{5}$.

$14 = 2 \cdot 7, \frac{2 \cdot 21}{3}, \frac{28}{2}, \frac{2 \cdot 35}{5}$ (read, $14 = 2 \cdot 7$'s, $\frac{2}{3}$ of 21).

$21 = 3 \cdot 7, \frac{3 \cdot 14}{2}, \frac{3 \cdot 28}{4}, \frac{3 \cdot 35}{5}$.

$28 = 4 \cdot 7, 2 \cdot 14, \frac{4 \cdot 21}{3}, \frac{4 \cdot 35}{5}$.

$35 = 5 \cdot 7, \frac{5 \cdot 14}{2}, \frac{5 \cdot 21}{3}, \frac{5 \cdot 28}{4}$.

4. Practice observing the units and writing the comparisons until you can do so readily.

5. Practice imaging the units and thinking the comparisons.

6. Find the ratio 2 as many times as you can; the ratio 3; the ratio $\frac{3}{2}$; the ratio $\frac{4}{3}$; the ratio $\frac{5}{2}$; the ratio $\frac{5}{3}$.

7. Image and name the units that have the ratio 2; that have the ratio 3; that have the ratio $\frac{3}{2}$; that have the ratio $\frac{1}{5}$; that have the ratio $\frac{3}{5}$.

Problems. — 1. If you pay \$20 for board for 28 dy., what ought you to pay for 14 dy.? for 7 dy.? for 21 dy.? for 35 dy.?

2. 6 equals $\frac{1}{2}$ doz. 12 equals how many half doz.? 18? 24?

3. 25 equals $\frac{1}{4}$ of 100. 20 equals what part of $\frac{1}{4}$ of 100? 15 equals what part? 10 equals what part? 5 equals what part?

4. If 20 is $\frac{1}{5}$, 16 equals what part of $\frac{1}{5}$? 12 equals what part? 8 equals what part? 4 equals what part?

5. 15 ft. equal 5 yd. 12 ft. equal what part of 5 yd.? 3 ft. equal what part? 9 ft. equal what part? 6 ft. equal what part?

6. If 3 equals $\frac{1}{2}$, 6 equals what? 9? 12? 15?

7. 15 costs \$20. What part of \$20 ought 12 to cost? 9? 6? 3?

8. If 15 equals $\frac{1}{2}$ of some number, 10 equals what part of the $\frac{1}{2}$? 5 equals what part? 20 equals what part? 25 equals what part?

9. 14 equals 2·7's. 28 equals how many 2·7's?

10. 7 costs 5¢. 35 will cost how many 5¢?

11. \$15 equals the cost of 18. What part of \$15 equals the cost of 6? of 12? of 24? of 30?

12. 10 equals 2·5's. 20 equals how many 2·5's?

13. 16 oz. equal 1 lb. 12 oz. equal what? 8 oz.? 4 oz.? 20 oz.?

14. Make sentences like this: $\frac{5}{4}$ is the ratio of 35 to 28.

15. $\frac{5}{3}$ is the ratio of what to 12? of what to 18?

What 5 equal units in 35? in 30? in 20? in 25? in 15?

Practice reading results at sight.

$\frac{30}{5}$	$\frac{35}{5}$	$\frac{20}{5}$	$\frac{15}{5}$	$\frac{25}{5}$
$\frac{2\cdot35}{5}$	$\frac{2\cdot30}{5}$	$\frac{2\cdot20}{5}$	$\frac{2\cdot25}{5}$	$\frac{2\cdot15}{5}$
$\frac{5\cdot21}{3}$	$\frac{5\cdot18}{3}$	$\frac{5\cdot15}{3}$	$\frac{5\cdot12}{3}$	$\frac{5\cdot9}{3}$

Image five of each with the sum. Practice.

7 6 5 4 3

1. If 12 min. is the time it will take 2 men to unload a wagon, what is the time it will take 1 man?

2. In $\frac{1}{2}$ hr. 3 girls can do how many times as much work as 1 girl can do in the same time?

3. If 12 represents a blackboard that 1 girl can clean in 5 min., what will represent a blackboard that 3 girls can clean in the same time?

4. If 4 boys do a piece of work for which 20¢ is paid, how much should each boy receive? Why?

5. If bananas are sold for 8¢ a dozen, 9 bananas will cost how much? 15 will cost how much?

6. If 1 lb. of raisins costs 16¢, what will $\frac{3}{4}$ lb. cost?

7. If $\frac{4}{5}$ lb. of raisins costs 16¢, what will 1 lb. cost?

8. At 16¢ a yard, what will $1\frac{1}{4}$ yd. of dress-lining cost?

9. At 5¢ a ride, how many rides can you take for 25¢?

10. Alice had 20¢. She spent $\frac{3}{5}$ of it. The money she had left equals what part of the money she spent?

11. A boy agreed to hoe a garden for 35¢. He hoed $\frac{4}{5}$ of it. What ought he to receive?

12. 7 equals what part of 28? If 28 represents $\frac{4}{5}$ of a garden, 7 represents what part of it?

13. A merchant has two pieces of curtain goods, which he wishes to cut into 6-yd. lengths. There are 30 yd. in one piece and 18 in the other. Into how many 6-yd. pieces can each be cut? What is the ratio of 3·6 yd. to 5·6 yd.? of 18 yd. to 30 yd.? of 30 to 18?

14. 24 equals $\frac{4}{5}$ of the length of a line. What equals $\frac{1}{5}$ of its length? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{5}$?

Ratios. — See method of work, page 88.

60	55	50	45	40
48	44	40	36	32
36	33	30	27	24
24	22	20	18	16
12	11	10	9	8

1. 48 yd. cost \$16. What was the cost of 24 yd.? of 36 yd.? of 12 yd.? of 60 yd.?

2. What unit is twice as large as 22? What unit equals $\frac{3}{2}$ of 22? $\frac{5}{2}$ of 22? $\frac{1}{2}$ of 22?

3. 50¢ equals \$$\frac{1}{2}$. 40¢ equals what part of \$$\frac{1}{2}$? 20¢ equals what part? 30¢ equals what part?

4. 9 in. equal $\frac{3}{4}$ ft. 18 in. equal how many $\frac{3}{4}$ ft.? 27 in.? 36 in.? 45 in.?

5. 36 in. equal 1 yd. 27 in. equal part of 1 yd.? 18 in.? 45 in.? 9 in.?

6. 8 costs \$5; 16 costs what? 24? 32? 40?

7. 24 equals $\frac{3}{2}$ of what? 32 equals $\frac{3}{4}$ of what? $\frac{4}{3}$ is the relation of what to 32?

8. 9 sq. ft. equal 1 sq. yd. 27 sq. ft. equal how many square yards? 3 is the relation of what to 9?

9. Make sentences similar to the following: If the cost of $1\frac{1}{4}$ yd. of ribbon is 45¢, the cost of 1 yd. is 36¢; of $\frac{3}{4}$ yd., 27¢. If an acre of land sells for \$55, $\frac{4}{5}$ of an acre sells for \$44.

What five equal units in 60? in 55? in 50? in 45? in 40?

Read results at sight. Practice.

$\frac{40}{5}$	$\frac{45}{5}$	$\frac{50}{5}$	$\frac{55}{5}$	$\frac{60}{5}$
$\frac{2\cdot40}{5}$	$\frac{2\cdot45}{5}$	$\frac{2\cdot50}{5}$	$\frac{2\cdot55}{5}$	$\frac{2\cdot60}{5}$
$\frac{3\cdot40}{5}$	$\frac{3\cdot45}{5}$	$\frac{3\cdot50}{5}$	$\frac{3\cdot55}{5}$	$\frac{3\cdot60}{5}$
$\frac{5\cdot24}{3}$	$\frac{5\cdot27}{3}$	$\frac{5\cdot30}{3}$	$\frac{5\cdot33}{3}$	$\frac{5\cdot36}{3}$

Image five of each of the following with the sum. Practice.

8 9 10 11 12

Make sentences like the following: $\frac{4}{5}$ is the ratio of 40 to 50; \$40 will support a family $\frac{4}{5}$ as long as \$50.

36	30	24	18	12
30	25	20	15	10
24	20	16	12	8
18	15	12	9	6
12	10	8	6	4
6	5	4	3	2

Ratios. — See method of work, pages 88, 108.

1. If we call the largest unit in a column 1, what should we call each of the other units in the column?

2. Compare $\frac{1}{2}$ with each of the other parts. Compare $\frac{1}{3}$ with each.

3. Compare each part with each of the other parts.

4. If the smallest unit is 6, how many 6's in each of the other units?

5. Name the greatest exact measure of two or more units.

Ex.: 12 is the greatest exact measure of 12, 24, 36.

6. Make applied problems, thus: If $48¢$ will buy x doz., $36¢$ will buy how many $\frac{1}{4}$ of x doz.?

7. Make sentences similar to the following: If 15 tons last 11 mo., 10 tons will last $\frac{2 \cdot 11}{3}$ mo.; 20 tons, $\frac{4 \cdot 11}{3}$ mo.; and 25 tons, $\frac{5 \cdot 11}{3}$ mo. If $\$x$ buys 12, $\frac{2 \cdot \$x}{3}$ will buy 8; $\frac{4 \cdot \$x}{3}$, 16; and $\frac{5 \cdot \$x}{3}$, 20.

Practice reading at sight.

$$\frac{12}{6} \quad \frac{18}{6} \quad \frac{24}{6} \quad \frac{30}{6} \quad \frac{36}{6}$$

$$\frac{12}{2} \quad \frac{18}{2} \quad \frac{24}{2} \quad \frac{30}{2} \quad \frac{36}{2}$$

$$\frac{12}{3} \quad \frac{18}{3} \quad \frac{24}{3} \quad \frac{30}{3} \quad \frac{36}{3}$$

$$\frac{2 \cdot 12}{3} \quad \frac{2 \cdot 18}{3} \quad \frac{2 \cdot 24}{3} \quad \frac{2 \cdot 30}{3} \quad \frac{2 \cdot 36}{3}$$

$$\frac{5 \cdot 12}{6} \quad \frac{5 \cdot 18}{6} \quad \frac{5 \cdot 24}{6} \quad \frac{5 \cdot 30}{6} \quad \frac{5 \cdot 36}{6}$$

Practice imaging six of each of the following with the sum. Practice pronouncing sums.

2 3 4 5 6

Problems. — 1. Cut rectangles to represent 1 and 2. If you call the smaller 16, what do you call the larger? if you call the smaller 15? if you call it 12? if you call it 9? if you call it 6?

2. Cut rectangles equal to $\frac{5}{6}$ of the rectangle you call 1. If you call the larger 36, what do you call the smaller? if you call the larger 30? if you call it 24? if you call it 18? if you call it 12?

3. If Fred earns $6¢$ while Joseph earns $5¢$, how much does Joseph earn while Fred earns $24¢$?

4. If Alice can walk 36 rd. and Julia 30 in a certain time, in $\frac{5}{6}$ of this time each can walk how far?

5. When butter was 24¢ a pound and lard 18¢, a lady paid 12¢ for butter and 9¢ for lard. How much of each did she buy?

6. If the car fare for 24 mi. is x¢, the car fare for 36 mi. is how much?

7. If the fare for 24 mi. is x¢, how many miles can you travel for $\frac{3}{2}$ of x¢?

Ratios. — See method of work, pages 88, 108.

72	66	60	54	48	42
60	55	50	45	40	35
48	44	40	36	32	28
36	33	30	27	24	21
24	22	20	18	16	14
12	11	10	9	8	7

Practice reading the following at sight:

$\frac{42}{2}$ $\frac{48}{2}$ $\frac{54}{2}$ $\frac{60}{2}$ $\frac{66}{2}$ $\frac{72}{2}$

$\frac{42}{3}$ $\frac{48}{3}$ $\frac{54}{3}$ $\frac{60}{3}$ $\frac{66}{3}$ $\frac{72}{3}$

$\frac{2\cdot42}{3}$ $\frac{2\cdot48}{3}$ $\frac{2\cdot54}{3}$ $\frac{2\cdot60}{3}$ $\frac{2\cdot66}{3}$ $\frac{2\cdot72}{3}$

$\frac{42}{6}$ $\frac{48}{6}$ $\frac{54}{6}$ $\frac{60}{6}$ $\frac{66}{6}$ $\frac{72}{6}$

$\frac{5\cdot42}{6}$ $\frac{5\cdot48}{6}$ $\frac{5\cdot54}{6}$ $\frac{5\cdot60}{6}$ $\frac{5\cdot66}{6}$ $\frac{5\cdot72}{6}$

Picture columns of six of each. Practice.

7 8 9 10 11 12

Picture columns and sums. Practice pronouncing sums.

Problems. — 1. If a sack of flour weighs 50 lb., $\frac{4}{5}$ of a sack will weigh how many pounds?

2. What is the ratio of the amount 54¢ will buy to the amount 45¢ will buy?

3. At 4¢ a yard, how much ribbon can you buy for 54¢?

4. What is the ratio of 1 sq. yd. to 54 sq. ft.?

5. If 72 represents the work that 12 men will do in an hour, what represents the work that 4 men will do in an hour?

Let each pupil write one problem involving some of the above ratios. Pupil read problem once. Other pupils write answer. Carefully note degree of accuracy and rapidity in order to adapt the work to the pupil.

Ratios. — If you do not already know the ratios of the following units, learn them. Continue the work until you know the relations thoroughly.

2 4 6 8 10 12 14 16 18 20 22 24
1 2 3 4 5 6 7 8 9 10 11 12

2 is the ratio of 2 to 1, of 4 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
2 4 6 8 10 12 14 16 18 20 22 24

$\frac{1}{2}$ is the ratio of 1 to 2, of 2 to 4, etc.

3 6 9 12 15 18 21 24 27 30 33 36
1 2 3 4 5 6 7 8 9 10 11 12

3 is the ratio of 36 to 12, of 33 to 11, etc.

1 2 3 4 5 6 7 8 9 10 11 12
3 6 9 12 15 18 21 24 27 30 33 36

$\frac{1}{3}$ is the ratio of 12 to 36, of 11 to 33, etc.

4 8 12 16 20 24 28 32 36 40 44 48
1 2 3 4 5 6 7 8 9 10 11 12

4 is the ratio of 48 to 12, of 44 to 11, etc.

1 2 3 4 5 6 7 8 9 10 11 12
4 8 12 16 20 24 28 32 36 40 44 48

$\frac{1}{4}$ is the ratio of 12 to 48, of 11 to 44, etc.

5 10 15 20 25 30 35 40 45 50 55 60
1 2 3 4 5 6 7 8 9 10 11 12

5 is the ratio of 60 to 12, of 55 to 11, etc.

1 2 3 4 5 6 7 8 9 10 11 12
5 10 15 20 25 30 35 40 45 50 55 60

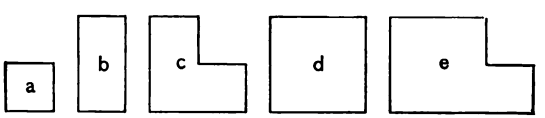
$\frac{1}{5}$ is the ratio of 12 to 60, of 11 to 55, etc.

6 12 18 24 30 36 42 48 54 60 66 72
1 2 3 4 5 6 7 8 9 10 11 12

6 is the ratio of 72 to 12, of 66 to 11, etc.

1 2 3 4 5 6 7 8 9 10 11 12
6 12 18 24 30 36 42 48 54 60 66 72

$\frac{1}{6}$ is the ratio of 12 to 72, of 11 to 66, etc.



Ratios. — 1. Observe the units and tell all you can about their relations.

2. Compare *b* with each of the other units.

3. Compare each of the other units with *b*.

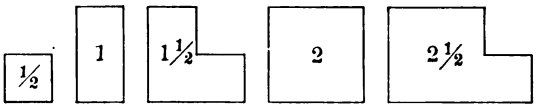
4. If *x* is the cost of *b*, what is the cost of each of the other units? *Ex.:* $\frac{x}{2}$ is the cost of *a*.

5. If *b* represents 8 yd. of carpet, what number of yards does each of the other units represent?

6. If 24 is the cost of *b*, what equals the cost of each of the other units?

7. If *b* is 1, each of the other units is what?

8. Make sentences like this: If *c* is worth \$7, *a* is worth $\$\frac{7}{3}$ (read, $\frac{1}{3}$ of \$7).



1. Tell all you can about the relation of the units.

2. Make sentences like this: The sum of 1 and $1\frac{1}{2} = 2\frac{1}{2}$.

3. Show me $\frac{1}{2}$ of the 1. How many $\frac{1}{2}$ of 1 do you find in each?

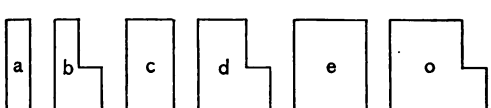
4. Compare the 1 with each of the other units. *Ex.:* $1 = 2$ times $\frac{1}{2}$, $\frac{2}{3}$ of $1\frac{1}{2}$, etc.

5. Compare $1\frac{1}{2}$ with each of the other units; compare 2 with each; compare $2\frac{1}{2}$ with each.

6. Make sentences like this: If the $1\frac{1}{2}$ is worth \$9, the $\frac{1}{2}$ is worth $\$\frac{9}{3}$.

7. Select solids having the same ratios.

Study by the methods suggested above.



1. Draw these units. Tell all you can about them.

2. Compare *a* with the other units.

3. Compare each of the other units with *a*.

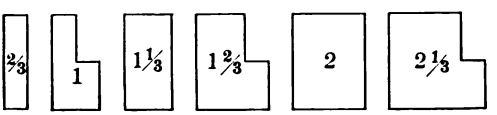
4. If *a* is 1, what is each of the other units? if *c* is 1, what is each? if *d* is 1? if *e* is 1? if *o* is 1? if *b* is 1?

5. Call a $\frac{2}{3}$ and name the other units.

6. Image the units and think $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$.

7. Make sentences like this: If c represents 16 sq. yd., $\frac{3}{4}$: $\frac{1}{4}$ $\frac{6}{}$ equal the number of square yards represented by b.

8. Pupils question one another, thus: If d will cover x sq. ft., c will cover what part of x sq. ft.?



1. Tell all you can about these units.

2. Make sentences like this: The sum of 1 and $1\frac{1}{3} = 2\frac{1}{3}$.

3. Show me $\frac{1}{3}$ of the 1. How many $\frac{1}{3}$ of 1 in each unit?

4. Compare 1 with each of the other units. Image and compare.

5. Compare $1\frac{1}{3}$ with each. Image and compare.

6. Compare $1\frac{2}{3}$ with each. Image and compare.

7. Compare 2 with each. Image and compare.

8. Work until you can express the above relations readily.

9. Make sentences like this: If 1 lb. of butter costs 18¢, $1\frac{1}{3}$ lb. will cost $\frac{4}{3}$: $\frac{18}{}$¢.

1. Select solids having the same ratios as the rectangles and name them $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$.

2. Study the relations by the methods suggested above.

3. Make applied problems. *Ex.:* If $1\frac{1}{3}$ tons of coal cost \$8, the cost of 1 ton is how many $\frac{1}{4}$ of \$8?

1. From a ribbon $2\frac{2}{3}$ yd. long, $1\frac{1}{3}$ yd. were cut. How much remained? $2\frac{2}{3}$ yd. are how much more than $1\frac{1}{3}$ yd.? If to $1\frac{1}{3}$ yd. $1\frac{1}{3}$ yd. be added, what is the sum? What is the ratio of $2\frac{2}{3}$ yd. to $1\frac{1}{3}$ yd.? of $1\frac{1}{3}$ yd. to $2\frac{2}{3}$ yd.?

2. I bought $2\frac{1}{2}$ lb. of brown sugar and $1\frac{1}{2}$ lb. of white. How much in all?

3. A square table is $1\frac{1}{3}$ by $1\frac{1}{3}$. What is its perimeter?

4. A square table is $1\frac{1}{2}$ by $1\frac{1}{2}$. What is its perimeter?

5. A rectangle is $1\frac{1}{2}$ by 2. What is its perimeter?

6. $\frac{1}{2}$ yd. of lace will make a collar. 2 yd. will make how many collars?

7. If $1\frac{1}{3}$ tons of hay cost \$12, how much does 1 ton cost?

8. Draw a rectangle having the ratio of $1\frac{2}{3}$ to $\frac{3}{3}$.

9. What is the ratio of the area of a paving stone containing $1\frac{2}{3}$ sq. yd. to one containing 1 sq. yd.?

10. $2\frac{1}{3}$ yd. will make a cloak. At $x\not{c}$ a yard, how many $\frac{1}{3}$ of $x\not{c}$ will it cost?

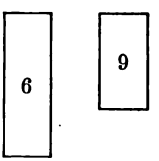
11. If 1 sack of sugar costs \$8, what will $1\frac{1}{2}$ sacks cost?

12. What is the ratio of the work a man can do in $2\frac{1}{2}$ hr. to the work he can do in $\frac{1}{2}$ hr.? to the work he can do in 1 hr.?

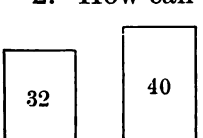
13. If \$10 will pay a man's rent for $\frac{1}{3}$ mo., how much will pay it for $2\frac{1}{3}$ mo.?

14. What is the ratio of $2\frac{1}{3}$ to $\frac{3}{8}$?

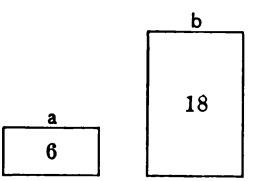
Exact Measures. — 1. What units are exact measures of 6 and 9? How many 1's in 6? in 9? How many 3's in each? What is the largest exact measure of 6 and 9? 3 equals what part of 6? 9 equals how many $\frac{1}{2}$ of 6? Why? 3 equals what part of 9? 6 equals how many $\frac{1}{3}$ of 9? Why? Before finding that 9 equals $\frac{3}{2}$ of 6, each unit had to be measured by what? If 6 is used as a measure, how many are found in the 6? in the 9? If 9 is used as the measure, how many are found in the 9? What part of this measure is there in the 6?



2. How can the relation of 32 and 40 be found? What is the name of the largest measure common to both? How many 8's in each? 8 equals what part of 32? 40 equals how many $\frac{1}{4}$ of 32? 8 equals what part of 40? 32 equals how many $\frac{1}{5}$ of 40?



3. Suppose the cost of b is given, how can the cost of a be found? Could you tell to what part of the cost of b the cost of a is equal without comparing the size of a with that of b? In making the comparisons, what must be used?



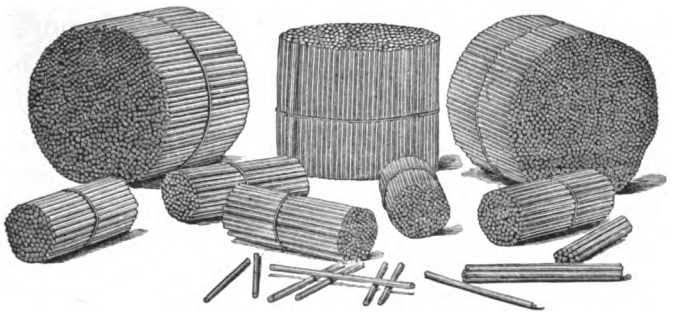
4. How many exact measures can you find of the units a and b? Have you found four? What are they?

5. What is the largest common measure of a and b? How many 6's in a? how many in b? a equals what part of b? b equals how many times a?

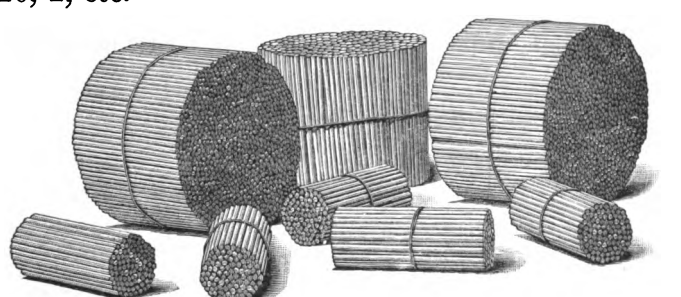
6. If we say that a equals $\frac{6}{18}$ of b, what is the common measure? If we say that b equals $\frac{18}{6}$ of a, what is still the common measure?

Ratios. — 1. What is the ratio of a to b? of b to a?

a	b	a	b
1 lb.	2 lb.	$1\frac{1}{3}$ yd.	2 ft.
5¢	1 dime	15¢	2 dimes
1 lb.	8 oz.	1 qt. 1 pt.	2 pt.
1 gal.	2 qt.	$\frac{3}{4}$	$\frac{1}{2}$
$1\frac{1}{2}$	2	20	10
16	24	52 wk.	39 wk.
24	36	1 yd. 1 ft.	1 yd.
6 in.	$1\frac{1}{2}$ ft.	1 ft.	9 in.
$2\frac{1}{2}$ lb.	1 lb.	$\frac{6}{9}$	$\frac{1}{3}$



Ratios. — Have pupils handle units of thousands, hundreds, tens, and ones. Teach thoroughly the comparison of 1000, 100, 10, 1, then of 2000, 200, 20, 2, etc.



Pupils select units of thousands and hundreds and tell what they have.

Ex.: This unit is the sum of 2 thousand 3 hundred (showing the thousands and hundreds before uniting them), or 2300.

Pupils select units, show class, and tell what they have. Teacher write names of units on the blackboard.

Ex.: Pupil selects 1 thousand and 4 hundred, shows them to class, unites them, and says: "This unit is the sum of 1 thousand 4 hundred, or 1400." Teacher writes 1400 on the blackboard.

Teacher write names of units on the blackboard and pupils find units.

Ex.: Teacher writes 3200. Pupil finds 3 thousand 2 hundred and tells what he has.

Teacher select units, tell what she has, and a pupil write on the blackboard.

Ex.: Teacher selects 4 thousand 1 hundred and says: "4 thousand 1 hundred." Pupil writes.

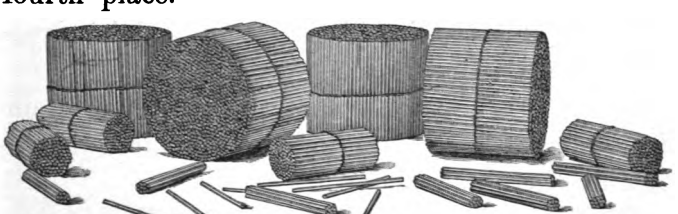
Pupil select and other pupils write.

Teacher show units and pupils write names on paper.

Pupils tell what they have written.

Ex.: The first unit is 4500. Teacher writes name of unit on the blackboard and pupils find unit.

Teacher write 2500. Show me the unit represented by the figure in the third place; in the fourth place.



Place bundles of thousands, hundreds, tens, and ones where they can be handled. In this, as in all the work,

provide for the union of perception and expression. If full opportunity is given, the selective action of the mind through eye and hand will become almost identical. So adapt the work to the pupil that it can be spirited, that quick movements will be possible. Ease and rapidity in moving, seeing, and handling are both effect and cause of ease and rapidity in mental action.

Under right conditions the notation and numeration accompany the perception of the relations just as naturally as the terms lemon or squirrel are related to these objects. (See Teachers' Book, p. 124.)

Do not try to exhaust a topic with young children. Seek, rather, to induce the mental attitude which welcomes returning to it from time to time.

Find the following units:

40	201
1001	102
240	222
2002	3040
1207	1050

Problems. — 1. A 30-in. string will reach how many times as far as a 2-in. string?

2. 4 30-in. strings will reach how many times as far as 4 2-in. strings?

3. If 3 boys mow a lawn for $x¢$, each receives what part of $x¢$? Why?

4. If 1 boy can mow the lawn in 1 hr., 3 boys can mow it in what part of 1 hr.?

5. At $40¢$ a dozen. 10 oranges cost what part of $40¢$?

6. If 3 oranges cost $10¢$, what is the price per dozen?

7. A 12-qt. can holds how many $\frac{1}{2}$ gal.?

8. At $2¢$ each, how many papers must a boy sell to receive $10¢$? If he sells $\frac{1}{5}$ of 10 papers, how many cents does he receive?

9. John had $10¢$ and lost $\frac{1}{5}$ of it. What is the ratio of the amount he has left to $10¢$?

10. A lady gave to Carrie 6 apples; to Margaret $\frac{3}{4}$ of 8 apples. What is the ratio of the number she gave to Carrie to the number she gave to Margaret?

11. The jelly that can be made of 4 peaches equals what part of the jelly that can be made of 10 peaches?

12. If $x¢$ is the cost of plastering 1 sq. ft., how many $9 \cdot x¢$ equal the cost of plastering 3 sq. yd.?

13. If the cost of paving $\frac{2}{9}$ of a walk is $m¢$, what is the cost of paving $\frac{2}{3}$ of it?

14. At $60¢$ a dozen, what part of a dozen oranges can be bought for $45¢$?

15. If there are 12 buttons on each shoe, how many buttons on 3 pairs of shoes?

16. What is the ratio of a 3-in. sq. to a 4-in. sq.?

17. At \$1 a bushel, what part of a bushel of apples can be bought for $60¢$?

18. At \$1 a bushel, how many pecks can you buy for $75¢$?

19. If 1 pt. of water weighs 1 lb., how much does 1 gal. weigh?

20. What is the perimeter of a paper 1 ft. by 7 in.?

21. A boy lost $\frac{1}{4}$ of his money and spent $\frac{1}{3}$ of the remainder. What part had he left?

22. If $\frac{3}{4}$ bu. of wheat make m lb. of flour, what part of m lb. will $\frac{1}{2}$ bu. make?

23. A square $\frac{1}{2}$ in. long equals what part of a square twice as long?

24. How many 3-in. squares can be cut from a paper 15 in. long and 3 in. wide? how many 2-in. squares?

25. The cost of $\frac{3}{8}$ of anything equals what part of the cost of $\frac{1}{2}$ of the same thing?

26. What is the ratio of $\frac{5}{6}$ to $\frac{1}{2}$? If $\frac{1}{2}$ yd. of cloth costs $30¢$, what is the cost of $\frac{5}{6}$ yd.?

27. What is the ratio of a square 2 by 2 to a square 4 by 4?

28. What is the ratio of a rectangle $\frac{1}{2}$ of a by $\frac{1}{2}$ of b to a rectangle a by b?

29. If I buy a $75¢$ book and give the clerk \$1, what does he give me in exchange?

30. If $\frac{2}{3}$ yd. of ribbon costs $40¢$, what is the cost of 1 yd.?

31. The cost of $\frac{2}{3}$ yd. of ribbon equals what part of the cost of $1\frac{2}{3}$ yd.?

32. What is the ratio of $1\frac{1}{5}$ to $\frac{5}{5}$?

33. At $60¢$ a yard, what is the cost of $1\frac{1}{5}$ yd. of lace?

34. What is the ratio of a rectangle 3 ft. sq. to a rectangle 2 ft. sq.? If a rug 2 ft. sq. costs $20, what is the cost of a rug 3 ft. sq.?

35. A boy sold a ball for $\frac{1}{4}$ more than the cost. What was the ratio of the selling price to the cost?

36. A boy sold a top for $\frac{1}{3}$ more than the cost. What he sold it for equals how many thirds of the cost? If the cost of the top was 15¢, for how much did he sell it?

37. If John buys a sled for $3, and sells it for $\frac{1}{3}$ more than the cost, for how much does he sell it? How much does he gain? What is the ratio of the gain to the cost?

38. Fred bought apples for $5, and sold them for $\frac{1}{5}$ less than he paid. For what did he sell them?

39. A grocer bought berries at 12¢ a box, and sold them for 15¢. What was the ratio of the selling price to the cost? of the cost to the selling price?

40. What is the ratio of 1 to $\frac{2}{3}$?

41. After erasing $\frac{1}{3}$ of a line, the length of the remainder was 18 in. What was the length of the line at first?

42. Write and answer 5 questions similar to the 43d.

43. What is the ratio of $\frac{5}{5}$ to the difference between $\frac{5}{5}$ and $\frac{1}{5}$?

44. A boy gave away $\frac{1}{5}$ of his money and had 40¢ left. How much had he at first?

45. If a man earns \$27 a month, how long will it take him to earn \$54?

46. What is the ratio of the amount which can be bought for 50¢ to the amount which can be bought for 30¢?

47. What is the ratio of the money a man can earn in 21 dy. to the money he can earn in 35 dy.?

48. If a boy can walk x mi. in 44 min., what part of x mi. can he walk in 33 min.?

49. To the amount of cloth I have, 4 yd. must be added to make a dress. What is the difference between the amount needed for the dress and the amount I have?

50. To make a 25-in. belt, how much must be added to a piece of belting $1\frac{1}{2}$ ft. in length?

51. If a is 16 more than b, what must be added to b that the sum may equal a? The sum of b and what equals a? a less what equals b? a less 16 equals what?

52. If to m you add n, the sum equals x. What is the difference between x and m? x less m equals what? x is how much more than m? x is how much more than n?

53. The sum of c and d equals y. What can you find to be true from knowing this?

54. I have 17 yd. of braid. How many yards will be left after binding a rug 4 ft. long and 1 ft. wide?

55. The shadows cast by two poles at the same time of day are equal. What is the ratio of the poles?

56. One pole casts a shadow 17 ft.; another at the same time of day casts a shadow 3·17 ft. What is the relative length of the poles?

57. After 1 lb. of sugar was taken out of a bowl, 8 oz. remained. How many ounces were in the bowl at first?

58. How many \$19 will 6000 ft. of lumber cost at \$19 per thousand?

59. When flour is selling at \$3 per hundred, how many hundred can be bought for \$21?

60. At $\frac{1}{3}$ of a dollar a pound, how many pounds of coffee can you buy for \$1? for \$5?

61. At $\frac{1}{5}$ of a dollar each, how many handkerchiefs can you buy for \$1? for \$3?

62. A man took the sod from a strip of land 10 yd. long and 2 yd. wide. How many square yards did he take?

63. How many rectangles of different dimensions, each containing 20 sq. in., can you make? What are the dimensions of each?

64. How many rectangles of different dimensions can you make, each containing 20·2-in. sq.?

65. John had two dimes. He bought 15 ft. of fish line at 3¢ a yard. How many cents had he left?

66. At 20¢ a quire, what part of a quire of paper can you buy for 15¢?

67. Miss Smith spent \$5 in 20 dy. At the same rate, in what time would she spend \$1? The ratio of \$1 to \$5 equals the ratio of what to 20 dy.?

68. How many cords of wood at \$8 a cord will pay for $1\frac{1}{2}$ bbl. of flour at \$8 a bbl.?

69. A man sold a table for \$8 and by so doing lost $\frac{1}{3}$ of its value. What was its worth?

70. At the rate of 8 \$100 for 12 mo. work, how many \$100 are paid for 3 mo. work?

71. How many yards of cloth at \$2 a yard must be given for 6 dy. work at \$2 a day?

72. What is the ratio of the number of pints in a can to the number of quarts? of the number of quarts to the number of pints.

73. What is the ratio of the number of square feet in a floor to the number of square yards in it? of the number of square yards to the number of square feet?

74. What is the ratio of the number of 3's in 12 to the number of 6's in 12? of the number of 6's in 12 to the number of 3's in 12?

75. What is the ratio of the number of 10's in 40 to the number of 10's in 100? of the number of 10's in 100 to the number of 10's in 40?

76. What is the ratio of the number of $12\frac{1}{2}$'s in 75 to the number in 50? of the number in 50 to the number in 75?

77. A dressmaker can cut 6 dresses while her assistant cuts 2. How many can the assistant cut while the dressmaker is cutting 24?

78. If for 6¢ a man obtains the use of $1 for 1 yr., for 60¢ he obtains the use of how many dollars for the same time?

79. The ratio of 60¢ to 6¢ equals the ratio of what to $1?

80. How many apples at 3¢ apiece must be given in exchange for a dozen pears at 4¢ apiece?

81. If 16 lemons cost 32¢, what will 24 lemons cost?

82. The ratio of the line a to the line b is 5. What is the ratio of $\frac{1}{4}$ of the line a to $\frac{1}{4}$ of the line b? of $\frac{3}{4}$ of the line a to $\frac{3}{4}$ of the line b? of a line 3 times the length of a to a line 3 times the length of b?

83. What is the ratio of a line 2 yd. long to a line 2 ft. long? of a line 6 ft. long to a line 2 ft. long?

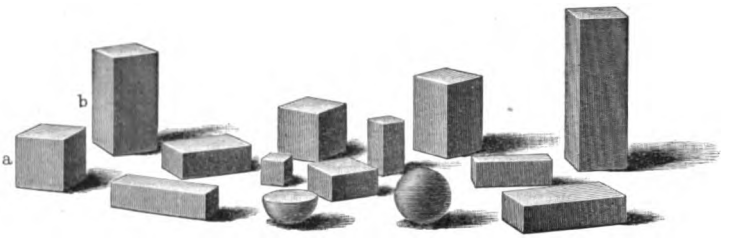
84. Draw 2 lines of equal length. What is the ratio of the lines? If you separate each of these lines into 7 equal parts, what is the ratio of the parts, each to each? If you separate them into 100 equal parts, what is their ratio?

PART II.

To attempt to force each class to acquire a given amount in a fixed time would be a grave error.[1] Assuming certain conditions, the time may be approximated. But beside the barriers which nature opposes to uniform achievement, there are infinite variations in the environment which help to decide what a pupil can and cannot do.

A child trained in a good kindergarten, or in a home which affords opportunity for fit and free activity, for daily contact with sympathetic and cultured minds, will have acquired a store of impressions, a power to observe and express, and a degree of self-control which may enable him to do in weeks what cannot be done in months by one whose faculties have never been wakened, whose curiosity has been checked before it blossomed into effort.

[1] "The seeds of the begonia taken from the same pod will germinate, some in a day, some at the end of a year, and some at various intermediate times, even when they are placed under the same external conditions."



Ratios. — 1. 2 is the ratio of b to a. What is the ratio of a to b?

2. Select all the solids that you can that have the ratio 2. Units that have the ratio 2 have what other ratio?

3. Observe units and tell any ratio you see. *Ex.:* $\frac{2}{3}$ is the ratio of this unit to that one.

4. State the ratio of the weight of the different solids. *Ex.:* $\frac{3}{4}$ is the ratio of the weight of this solid to the weight of that one.

5. State the ratio of the number of cubic inches in each. *Ex.:* $\frac{4}{5}$ is the ratio of the number of cubic inches in this solid to the number in that.

6. Give ratio of the cost of the solids based on the relative size. *Ex.:* $\frac{5}{6}$ is the ratio of the cost of this solid to the cost of that one.

7. Give ratio of the heights of different solids; of lengths.

8. Close eyes. Handle solids and guess at relative size. Infer relative weight. Try to determine the number of cubic inches in each solid by means of touch and muscular sense.

Give short exercises for several days in observing and stating the ratios between solids, surfaces, and lines, respectively.

1. Show by drawing lines or rectangles the ratios of the following units, thus: the ratio of 1 ft. to 6 in. is _____ ____.

2. What is the ratio of 1 ft. to 6 in.?

3. What is the ratio of 1 ft. to 4 in.? to 3 in.? to 2 in.?

4. What is the ratio of 2 ft. to 6 in.? of 1 yd. to 6 in.?

5. What is the ratio of 1 gal. to 2 qt.? to 1 qt.? to 3 qt.? to $1\frac{1}{2}$ gal.?

6. What is the ratio of 1 qt. to 1 pt.? of 1 qt. to 3 pt.? of 1 gal. to 1 pt.? of 1 gal. to 6 pt.?

7. What is the ratio of a dime to a nickel? of a dime to 15¢? of a dime to 25¢? of a nickel to $1\frac{1}{2}$ dimes?

8. What is the ratio of 1 yr. to 4 mo.? of 15 min. to 1 hr.? of 1 to $\frac{3}{4}$? of 1 to $\frac{2}{3}$? of 3 to $1\frac{1}{2}$? of $\frac{1}{2}$ to $\frac{1}{4}$? of $\frac{1}{2}$ to $\frac{1}{3}$? of $\frac{1}{2}$ to $\frac{5}{6}$? of 12 to 15? of 4 to 10? of 10 to 4? of 16 to 20? of 72 to 12? of 60 to 12? of 72 to 60?

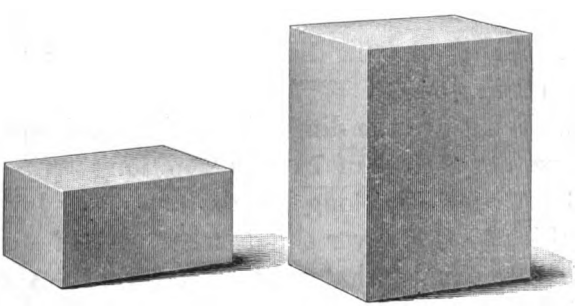
1. Make ten sentences like this: In 28 there are 4 7's. $\frac{1}{4}$ is the ratio of 7 to 28. 4 is the ratio of 28 to 7.

equal to 2 layers of 3 rows of 4 cubes, 2 in. long, are 3 · 2's by 4 · 2's by 2 · 2's.

6. Look at a 3-in. cube. Make mental picture of solids built of 3-in. cubes and express the dimensions in 3's.

7. In the volume of a room 12 ft. long, 10 ft. wide, and 8 ft. high, how many layers 1 ft. high? Each layer equals how many rows of how many cubic feet?

8. Make many sentences like this: The volume of a room 12 ft. long, 10 ft. wide, and 8 ft. high equals 8 layers of 10 rows of 12 cu. ft., or it equals 8 layers of 12 rows of 10 cu. ft. Review.



9. What is the ratio of a solid 5 by 4 by 3 to a solid 2 by 4 by 3?

10. Make sentences like this: $\frac{5}{2}$ is the ratio of a solid 5 by 4 by 3 to a solid 2 by 4 by 3.

11. Make sentences like this: 3 is the length of the largest cube that can be cut out of a solid 4 by 3 by 5.

1. Observe different surfaces of solids and tell the number of rows and the number of squares in each row, thus: This surface equals 3 rows of 4 sq. in.

2. Estimate the number of rows and the number of square feet in each of the blackboards. Measure.

3. Estimate the number of rows and squares in other surfaces. Measure.

4. A square 7 in. long equals how many rows of how many square inches?

5. Make sentences like this: A square whose length is 12, equals 12 rows of 12 squares.

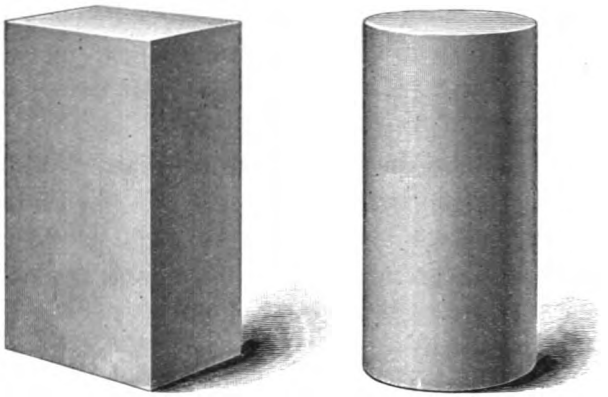
6. Observe surfaces and tell the length of the perimeter of each, thus: The perimeter of a surface 3 by 4 equals the sum of 2 3's and 2 4's, or 14. $2\,(3 + 4) = 14$.

7. Tell the perimeter of squares of different lengths. Review.

1. Tell the altitude and the dimensions of the base of different solids. *Ex.:* The altitude of *c* is 9 and its base is 3 by 4.

2. Make sentences like this: A solid whose base is 4 by 5 and altitude 7 equals 7 layers of 4 rows of 5 cubes.

3. Show me the upper base of different solids. Show me the lower base. Show me the bases of a cylinder. Show me the bases of this room.



c

4. Tell the perimeter of the base of different solids? *Ex.:* If the dimensions of the base of a solid are 3 by 4, its perimeter equals 14. 2 $(3 + 4) = 14$.

5. Show me the lateral surface of *c*; of the cylinder. How many parts in the lateral surface of *c*? of the cylinder?

6. How many walls in the room? How many parts in the lateral surface of the room? Point to the bases of the room; to the four parts of the lateral surface.

7. Show me the base of *c*. What is the length of the perimeter of the base of *c*?

8. Tell the length of the perimeter of the base of other solids.

9. Review.

1. Make a strip of paper 1 in. wide and bend it around c so that the ends of the strip meet. Measure the strip. It equals how many square inches?

2. If the altitude of c is 9 in., how many rows of 14 sq. in. in the lateral surface of c?

3. The altitude of a solid is 6; its base is 5 by 7. How many squares in one row of the lateral surface of the solid? How many rows of 24 sq. in the lateral surface of the solid?

4. Make sentences like this: There are 6 rows of 10 sq. in the lateral surface of a solid whose altitude is 6 and base 2 by 3.

5. How many rows of how many squares in the lateral surface of a cube whose length is 5?

6. Make sentences like this: There are 5 rows of 20 sq. in the lateral surface of a cube whose length is 5.

7. How many rows of 5 sq. in the sum of the bases of a cube whose length is 5?

8. Make sentences like this: The sum of the bases of a cube whose length is 5 equals 10 rows of 5 sq.

1. If the base of a room is 10 ft. by 12 ft., what is the number of square feet in one row in the lateral surface? $2\ (10 + 12) =$ the number.

If the altitude of the room is 8 ft., how many rows of 44 sq. ft. in the lateral surface of the

room? What is the number of square feet? $8 \cdot 2\ (10 + 12)$ = the number.

2. What equals the number of square feet in the lateral surface of a room 35 ft. long, 30 ft. wide, and 12 ft. high?

3. Make sentences like this: 12 · 130's equals the number of square feet in the lateral surface of a room 35 ft. long, 30 ft. wide, and 12 ft. high. $12 \cdot 2\ (30 + 35)$ = the number.

4. What equals the number of square yards in one row of the lateral surface of a room 8 yd. long and 9 yd. wide? If the room is 4 yd. high, what equals the number of square yards in the lateral surface? $4 \cdot 2\ (8 + 9)$.

5. What is the ratio of a surface 2 by 3 to a surface 4 by 3?

6. Make sentences like this: 2 is the ratio of a surface 4 by 3 to a surface 2 by 3.

7. Think of a surface and give its dimensions. Give the dimensions of a surface equal to $\frac{1}{2}$ the first.

8. Make sentences like this: A surface 3 by 4 equals $\frac{1}{4}$ of a surface 6 by 8.

9. Make sentences like this: 5 ft. is the length of the largest square that can be cut out of a rectangle 5 ft. by 7 ft.

1. What are the dimensions of 1 row of 5 sq. in.? of a rectangle containing 2 rows of 7 sq. in.?

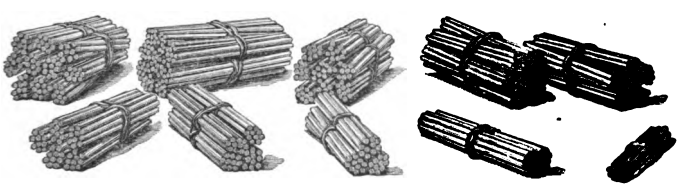
of 1 row of 3 · 2-in. sq.? of a rectangle containing 3 rows of 5 · 2-in. sq.?

2. What is the perimeter of 1 row of 8 sq. in.? of 2 rows of 8 sq. in.? of 3 rows of 5 · 2-in. sq.? of 2 rows of 7 sq. yd.? of 2 rows of 5 · 2-ft. sq.?

3. What are the dimensions of a rectangular solid consisting of 3 layers of 5 rows of 7 cu. in.?

4. How many cubic feet can you place against a wall of a room 8 ft. high and 9 ft. long?

5. If a box is 6 in. wide and 4 in. high, how long will it have to be to hold 24 · 2-in. cubes?



Make bundles of sticks consisting of 1 · 10, 2 · 10's, 3 · 10's, 4 · 10's, 5 · 10's, 6 · 10's, 7 · 10's, 8 · 10's, 9 · 10's, and 10 · 10's, and place them where they can be handled.

Ratios. — 1. Tell all the ratios that you can, using the names 10, 20, 30, etc.

2. Which units have the ratio 2? How many times have you found it repeated?

3. Which units have the ratio 3?

4. What is the ratio of 30 to 10? of 60 to 20? of 90 to 30?

5. Which units have the ratio 4? the ratio 5?

6. Which units have the ratio $\frac{1}{2}$? the ratio $\frac{1}{3}$? the ratio $\frac{1}{4}$? the ratio $\frac{1}{5}$? the ratio $\frac{1}{10}$? the ratio 10?

7. What units have the ratio $\frac{3}{2}$? $\frac{4}{3}$? $\frac{5}{3}$? $\frac{5}{4}$?

8. Which units have the ratio $\frac{2}{3}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{4}{5}$?

9. Picture units and review.

10. Continue to work with these units until the ratios are mastered.

Represent the units on the blackboard by rectangles, making the *ten* 10 in. by 1 in.

11. Think the ratio of each to each of the others.

1. The ratio of 20 to 10 equals the ratio of what to $\frac{1}{2}$?

2. The ratio of 40 to 60 equals the ratio of what to 6?

3. $\frac{7}{8}$ is the ratio of 7 to what? of 70 to what?

4. The ratio of 60 to 100 equals the ratio of 6 to what?

5. What is the ratio of the work required to pave 30 sq. yd. to the work required to pave 50 sq. yd.? 100 sq. yd.?

6. What is the ratio of the number of men required to pave 30 sq. yd. to the number required to pave 50 sq. yd.?

7. The ratio of the cost of paving 50 sq. yd. to the cost of paving 30 sq. yd. is what?

8. $\frac{3}{2}$ is the ratio of Frank's money to John's. John has 60¢. How much has Frank?

9. If a man's expenses are $30 a month, $70 will support him for what time?

10. When butter is bought for 20¢ and sold for a sum equal to $\frac{3}{2}$ of the cost, what is the selling price?

11. If butter is bought for 20¢ and sold for 30¢, the gain equals what part of the cost? What is the ratio of the selling price to the cost?

1. What is the ratio of 10 to 5? What, then, is the ratio of 70 to 5?

2. Make sentences like this: 7 2's is the ratio of 70 to 5.

3. What is the ratio of 10 to 2? of 80 to 10? of 80 to 2? The ratio of 80 to 2 is how many times the ratio of 10 to 2?

4. Make sentences like this: 8 5's is the ratio of 80 to 2.

5. What is the ratio of 10 to 4? What is the ratio of 70 to 4?

6. Make sentences like this: 7 $\frac{5}{2}$ is the ratio of 70 to 4?

7. What is the ratio of 10 to 6? of 40 to 6? of 70? of 90? of 100?

8. What is the ratio of 10 to 8? of 60 to 8? of 90? of 70? of 80? of 50?

1. What is the ratio of 100 to 50? of 400 to 50? of 600? of 500?

2. What is the ratio of 100 to 20? 600 equals how many times 100? Then what is the ratio of 600 to 20? of 400 to 20? of 800? of 500? of 700? of 900?

3. What is the ratio of 100 to 10? of 300? of 500? of 400? of 700? of 900?

4. What is the ratio of 100 to 40? of 600?

5. Make sentences like this: $6 \cdot \frac{5}{2}$ is the ratio of 600 to 40.

6. What is the ratio of 100 to 60? What is the ratio of 700 to 60?

7. Make sentences like this: $7 \cdot \frac{5}{3}$ is the ratio of 700 to 60.

8. What is the ratio of 100 to 30? of 400 to 30?

9. Make sentences like this: $4 \cdot \frac{10}{3}$ is the ratio of 400 to 30.

10. What is the ratio of 100 to 70? of 500 to 70? of 900?

11. Make sentences like this: $5 \cdot \frac{10}{7}$ is the ratio of 500 to 70?

Problems. — 1. A primer costs 10¢ and a pencil 5¢. What is the ratio of the cost of the primer to the cost of the pencil? If a geography costs 70¢, what is the ratio of its cost to the cost of the primer? to the cost of the pencil?

2. A paper of needles costs 10¢, a spool of thread 2¢, and a thimble 80¢. The ratio of the cost of the thimble to the cost of the thread is

how many times the ratio of the cost of the needles to the cost of the thread?

3. What is the ratio of a 10-in. line to an 8-in. line? of a line 6 times 10 in. to an 8-in. line? of a line 6 times 10 in. to a line 6 times 8 in.?

4. Fred drew 2 lines having the ratio 2. He then extended each line until it was three times as long as at first. What was then the ratio of the lines?

5. Draw 2 rectangles having the ratio 3. Make rectangles equal to $\frac{1}{2}$ of each. What is the ratio of these rectangles? Make rectangles 2 times as large as the first. What is their ratio?

6. What is the ratio of the amount of corn raised on a 100-acre farm to the amount raised on a 50-acre farm? of the amount raised on 400 acres to the amount raised on 50 acres?

7. A chain costs \$20, which equals $\frac{1}{5}$ of the cost of a watch. What is the cost of the watch? What is the cost of 6 such watches? The ratio of the cost of a \$100 watch to the cost of a \$20 chain is what? The ratio of 6 · \$100 to \$20 is what?

8. If the interest on \$100 for a year is \$$x$, what is the interest for the same time on \$400? \$700?

1. Draw 2 rectangles, one representing the work that 4 girls can do in a certain time, the other the work that 2 girls can do in the same time. What is the ratio of the rectangles?

2. If 4 girls can sweep a house in 15 min., what part of it can 3 girls sweep in 15 min.?

3. A stone 4 ft. by 3 ft. by 2 ft. contains how many cubic feet? How many square feet in the surface of this stone?

4. A man paid $\$2\frac{1}{3}$ for a hat and $\$4\frac{1}{2}$ for shoes. How much did he pay for both?

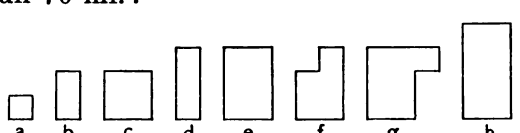
5. A piece of carpet 2 ft. sq. is worth $\$1$. How much is a piece 3 ft. sq. worth?

6. If $\frac{4}{5}$ lb. of butter is worth $20¢$, how much is 1 lb. worth?

7. A long ladder has 15 steps and a short one has 7. How many more steps has the long ladder than the short one?

8. Frank is 4 ft. 5 in. in height; Charles is 3 ft. 2 in. How much taller is Frank than Charles?

9. A train will run 100 mi. in how many sevenths of the time it will run 70 mi.? It will run 600 mi. in how many sevenths of the time it will run 70 mi.?



Ratios. — 1. Draw these units on the blackboard, making *a* a 6-in. sq.

2. Discover all the ratios you can.

3. Write the ratios that you see.

4. What units have the ratio 2?

5. Tell the names of other units that have the ratio 2. *Ex.:* 2 is the ratio of 1 to $\frac{1}{2}$; of 3 to $1\frac{1}{2}$; of $\frac{4}{3}$ to $\frac{2}{3}$.

6. What units in the set have the ratio 3? Give the names of other units that have the ratio 3.

7. Which units have the ratio 4? the ratio 5?

8. Which units have the ratio $\frac{1}{2}$? the ratio $\frac{1}{3}$? the ratio $\frac{1}{4}$?

9. Which units have the ratio $\frac{3}{2}$? which have the ratio $\frac{2}{3}$? which have the ratio $\frac{4}{3}$? the ratio $\frac{3}{4}$?

10. Cover the drawings and think the ratio of each to the others.

11. Practice until you have mastered thinking and expressing these ratios.

Problems. — 1. What is the ratio of $\frac{1}{2}$ to $\frac{1}{4}$? of $1\frac{1}{2}$ to $\frac{3}{4}$?

2. If $\frac{3}{4}$ yd. of ribbon costs $30¢$, what will $1\frac{1}{2}$ yd. cost? The cost of $\frac{1}{2}$ yd. equals what part of the cost of $1\frac{1}{2}$ yd.?

3. $\frac{1}{2}$ bu. of nuts is worth $50¢$. What are $1\frac{1}{2}$ bu. worth?

4. What is the perimeter of a square lamp-stand $\frac{3}{4}$ ft. on each side?

5. If a man's profits are \$500 in $1\frac{1}{2}$ yr., what are they in 3 yr.?

6. If $1\frac{1}{2}$ doz. eggs are worth $x¢$, what part of $x¢$ are 1 doz. worth?

7. A florist sold $\frac{2}{3}$ of his roses and had 5 left. How many had he at first?

8. John received 15¢ for working $\frac{2}{3}$ hr. At the same rate, how much should Charles, who worked $1\frac{1}{3}$ hr., receive?

9. $1\frac{1}{4}$ gal. of milk are worth how many times as much as $\frac{3}{4}$ gal.?

10. What is the ratio of $2\frac{1}{3}$ to $\frac{3}{3}$?

11. At $30 a month, what is the rent of a house for $\frac{2}{3}$ mo.?

1. What is the ratio of a 2-in. sq. to a 1-in. sq.? of a 2-in. sq. to a $\frac{1}{2}$-in. sq.? of a 2-in. sq. to a 4-in. sq.?

2. If the ratio of two gardens is 5, what is the ratio of $\frac{1}{2}$ of the larger to $\frac{1}{2}$ of the smaller?

3. How many strips of carpet $1\frac{1}{2}$ yd. long can be cut from a strip of carpet $7\frac{1}{2}$ yd. long?

4. John received $1.48 for 2 bu. of walnuts. What should he receive for $1\frac{1}{2}$ bu.? for 3 bu.? for 2 pk.? for 5 bu.?

5. How many 2-qt. boxes can be filled from 3 pk. of berries?

6. How many half-peck measures of meal are there in a sack containing $1\frac{1}{2}$ bu.?

7. A boy's steps are 2 ft. in length. How many steps will he take in crossing a bridge 6 yd. in length?

8. How many 18-in. sticks can be made from 4 laths, each lath 1 yd. long?

9. How many boards 1 ft. sq. in a plank 10 ft. long and 12 in. wide?

10. A 12-ft. plank is damaged 10 in. at one end and 8 in. at the other. How long will the plank be, if the damaged portions are sawed off?

Ratios. — 1. What is the ratio of a solid 2″ (2 in.) by 3″ by 5″ to a solid 1″ by 3″ by 5″?

2. What is the ratio of a solid 7″ by 3″ by 5″ to a solid 1″ by 3″ by 5″?

3. What is the ratio of a solid 2″ by 3″ by 7″ to a solid 4″ by 3″ by 7″?

4. What is the ratio of a solid 1″ by 4″ by 5″ to a solid $\frac{1}{2}$″ by 4″ by 5″?

5. What is the ratio of a solid 1 by 7 by 9 to a solid $\frac{1}{2}$ by 7 by 9?

6. What is the ratio of a solid a by b by c to a solid $\frac{a}{2}$ by b by c?

7. What is the ratio of a solid m by o by x to a solid $\frac{m}{4}$ by o by x?

1. Draw a rectangle 1″ by 2″; another $\frac{1}{2}$″ by 2″. What ratios do you see? What are the ratios of the areas? of the lengths? of the widths?

2. Draw a rectangle 1″ by 3″; another $\frac{1}{4}$″ by 3″. What ratios do you see?

3. What is the ratio of a rectangle 1″ by 5″ to a rectangle $\frac{1}{2}$″ by 5″?

4. What is the ratio of a rectangle 1″ by 5″ to a rectangle $\frac{1}{4}$″ by 5″?

5. What is the ratio of a rectangle a by b to a rectangle $\frac{a}{2}$ by b? to a rectangle $\frac{a}{4}$ by b?

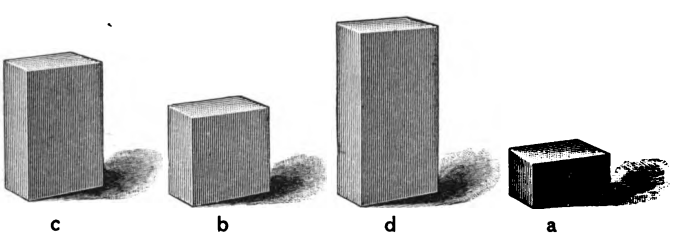
6. What is the number of square inches in a rectangle 2″ by 7″? What part of 14 equals the number of square inches in a rectangle 1″ by 7″?

7. What is the number of square inches in a rectangle 1″ by 12″? What part of 12 equals the number of square inches in a rectangle $\frac{1}{2}$″ by 12″?

8. A rectangle equals 3 sq. in. and is 3 in. long. What is its altitude?

9. If a rectangle equals 1 sq. in. and is 3″ long, what is its altitude?

10. The area of a rectangle equals 1 sq. in. It is 6″ long. What is its altitude?



1. If d represent 1, what is represented by each of the other units?

2. Compare $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ and 1 with each other.

3. If d be called 100, what is each of the other units?

4. Compare each unit with the other three. *Ex.:* $25 = \frac{50}{2}$ (read, $\frac{1}{2}$ of 50), $\frac{75}{3}$, $\frac{100}{4}$.

5. Show me $\frac{1}{2}$ of 25. $\frac{1}{2}$ of 25 equals what part of each of the other units?

6. The sum of $12\frac{1}{2}$ and $12\frac{1}{2}$ equals what? What is the ratio of each unit to 25? to $\frac{1}{2}$ of 25? of \$1 to 25¢? to $\frac{1}{2}$ of 25¢?

7. 25 equals how many times $12\frac{1}{2}$? $12\frac{1}{2}$ equals what part of 25? What is the ratio of 25 to $12\frac{1}{2}$? of $12\frac{1}{2}$ to 25?

8. If there are $2 \cdot 12\frac{1}{2}$ in 25, how many in 50? in 75? in 100?

9. Compare $12\frac{1}{2}$ with each unit. Compare each unit with $12\frac{1}{2}$.

10. How many $12\frac{1}{2}$¢ in a quarter dollar? in a half dollar? in 75¢? in \$1?

11. Review.

Problems. — 1. At 25¢ a peck, what will a bushel of apples cost? At $\frac{1}{2}$ of 25¢ a peck, what part of a dollar will a bushel cost?

2. At \$1 a day, how many $\frac{1}{2}$ dollars does a man earn in 2 dy.? how many 25¢? how many $12\frac{1}{2}$¢?

3. At $12\frac{1}{2}$¢ a yard, how many yards of lawn can be bought for 25¢? how many, then, for \$1?

4. What is the rate per hour when \$1 is paid for 8 hr. work?

5. At \$1 for 8 hr., 75¢ will pay for how many hours' work?

6. At \$1 a day, what part of a day does a man work who receives 75¢?

7. When a man has earned \$100, how many \$25 will he have earned? how many halves of \$25? how many \$12½?

8. At 12½¢ a yard, how many yards of lace can you buy for 50¢? for \$2?

1. John lives 27 mi. north of the courthouse and Henry 27 mi. south. How far apart are their homes?

2. It was 5° below zero yesterday and it is 7° above to-day. What is the difference in the temperature of the two days?

3. Nettie has 4¢ more than a dime and Edna has 5¢ less than a dime. How many more cents has Nettie than Edna?

4. Two men start from the same place and travel in opposite directions at the rate of 6 mi. an hour. How far apart are they at the end of an hour?

5. A school contains 5 more girls than boys and there are 27 girls. How many boys are there?

6. There are 13 more sheep in a field than cows. If there are 37 sheep in the field, how many cows are there?

7. In an orchard there are 45 peach trees and 30 apple trees. How many more peach trees than apple trees?

8. If the cost of $\frac{1}{3}$ bbl. of flour is \$4, what is the cost of $\frac{1}{2}$ bbl.?

9. A yard of trimming is needed for a collar and $\frac{3}{8}$ yd. for cuffs. How much more is needed for the collar than for the cuffs?

10. One package weighs 3 lb. and another 4 times as much. How much do both weigh?

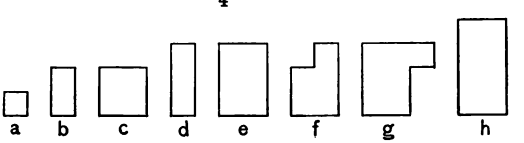
11. Mr. Smith earned 75¢ and his son $\frac{1}{3}$ as much. How much did they both earn?

12. If $\frac{3}{4}$ gal. of vinegar costs 50¢, what does $\frac{3}{8}$ gal. cost?

13. How many nickels will pay for a pound of coffee at 40¢ a pound? The *number* of dimes which will pay for the coffee equals what part of the *number* of nickels?

14. Fred earns $12\frac{1}{2}$¢ an hour and Leigh 25¢. What is the ratio of their earnings? If the wages of each were doubled, what would then be the ratio of their earnings?

15. What is the ratio of 1 bu. 3 pk. to 2 bu.? What is the ratio of $1\frac{3}{4}$ to 2?



Ratios. — 1. If *h* be called 1, each of the other units equals what part of 1?

2. If *a* is $12\frac{1}{2}$, each of the other units equals how many $12\frac{1}{2}$?

$12\frac{1}{2}$							
$12\frac{1}{2}$	$12\frac{1}{2}$						
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$					
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$				
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$			
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$		
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	
$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$	$12\frac{1}{2}$
100	$87\frac{1}{2}$	75	$62\frac{1}{2}$	50	$37\frac{1}{2}$	25	

3. Practice imaging and writing these combinations with their sums.

4. Make sentences similar to the following: The ratio of 100 to 100 is 1; to $87\frac{1}{2}$ is $\frac{8}{7}$, etc. The ratio of $8 \cdot 12\frac{1}{2}$ to $5 \cdot 12\frac{1}{2}$ equals the ratio of 100 to $62\frac{1}{2}$.

5. Select the units on the blackboard to which these names belong. *Ex.:* The name of f is $62\frac{1}{2}$, if a is $12\frac{1}{2}$.

6. Practice thinking the ratios. *Ex.:* $\frac{5}{3}$ is the ratio of $62\frac{1}{2}$ to $37\frac{1}{2}$. $\frac{5}{3}$ is the ratio of what to $37\frac{1}{2}$?

7. Practice writing the ratios of each to each of the others. *Ex.:* 5 is the ratio of $62\frac{1}{2}$ to $12\frac{1}{2}$; $\frac{5}{2}$, of $62\frac{1}{2}$ to 25; $\frac{5}{3}$, of $62\frac{1}{2}$ to $37\frac{1}{2}$; $\frac{5}{4}$, of $62\frac{1}{2}$ to 50; etc.

Problems. — 1. What is the ratio of the cost of 100 bu. of wheat to the cost of $12\frac{1}{2}$ bu.? of the

cost of 100 bu. to the cost of $4 \cdot 12\frac{1}{2}$ bu.? of the cost of 100 bu. to the cost of $62\frac{1}{2}$ bu.? to the cost of $87\frac{1}{2}$ bu.?

2. What is the ratio of $12\frac{1}{2}¢$ to \$1? The number of pounds of sugar I can buy for $12\frac{1}{2}¢$ equals what part of the number of pounds that I can buy for \$1? for $50¢$?

3. If 10 lb. of sugar can be bought for $62\frac{1}{2}¢$, how many fifths of 10 lb. can be bought for \$1?

4. Cloth bought at $37\frac{1}{2}¢$ was sold at $62\frac{1}{2}¢$. What was the ratio of the selling price to the cost? What was the ratio of the gain to the selling price?

5. If you buy for $\$\frac{3}{8}$ and sell for $\$\frac{5}{8}$, what is the ratio of the selling price to the cost? What part of \$1 do you gain? How many cents?

6. If you buy apples at \$1 a bushel and sell them for $87\frac{1}{2}¢$ a bushel, what is the ratio of the selling price to the cost? What is the ratio of the loss to the cost?

7. For $75¢$ we can buy how many times as much as for $37\frac{1}{2}¢$?

8. At $\$\frac{3}{8}$ each, how many yards of challis can be bought for $75¢$?

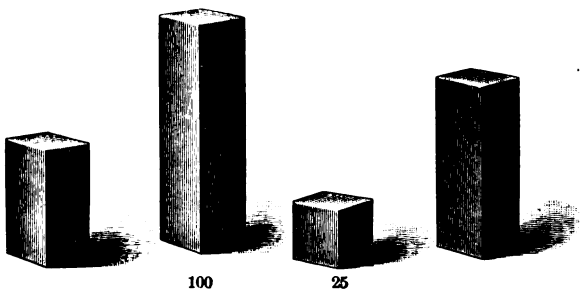
9. At $25¢$ a box, how many boxes of candy can you buy for \$1? at $12\frac{1}{2}¢$ a box?

10. $\frac{5}{8}$ is the ratio of the money Nettie has to \$1. How much has she?

11. At \$1 a yard, what part of a yard of velvet can be bought for $62\frac{1}{2}¢$? for $87\frac{1}{2}¢$?

12. What is the ratio of \$1 to $62\frac{1}{2}¢$? At $62\frac{1}{2}¢$ a yard, how many yards of velvet can be bought for \$1?

13. If $\frac{5}{8}$ of a yard of cloth costs $62\frac{1}{2}¢$, what will a yard cost?



Ratios. — 1. What is the ratio of 100 to 25? of 600 to 25? of 300? of 800? of 700? of 500?

2. What is the ratio of 100 to $12\frac{1}{2}$? of 300 to $12\frac{1}{2}$? of 700? of 500? 8 is the ratio of what to $12\frac{1}{2}$?

3. What is the ratio of 100 to $37\frac{1}{2}$?

4. If $\frac{8}{3}$ is the ratio of 100 to $37\frac{1}{2}$, what is the ratio of 500 to $37\frac{1}{2}$?

5. Make sentences like this: $\frac{8}{3}$ is the ratio of 100 to $37\frac{1}{2}$ and $5\cdot\frac{8}{3}$ is the ratio of 500 to $37\frac{1}{2}$.

6. What is the ratio of 700 to $37\frac{1}{2}$? of 900 to $37\frac{1}{2}$? of 1000? of 400?

7. What is the ratio of 100 to $62\frac{1}{2}$? $\frac{8}{5}$ is the ratio of what to $62\frac{1}{2}$? $\frac{5}{8}$ is the ratio of what to 100?

8. What is the ratio of 300 to $62\frac{1}{2}$?

9. Make sentences like this: $3\cdot\frac{8}{5}$ is the ratio of 300 to $62\frac{1}{2}$.

10. What is the ratio of 100 to $87\frac{1}{2}$? of 400 to $87\frac{1}{2}$?

11. Make sentences like this: $4\cdot\frac{8}{7}$ is the ratio of 400 to $87\frac{1}{2}$.

12. What is the ratio of \$1 to $37\frac{1}{2}¢$? of \$3? of \$5? of \$7?

13. Write many sentences like this: For \$5 you can buy $5\cdot\frac{8}{3}$ yd. of cloth at $37\frac{1}{2}¢$ a yard. For \$9 you can buy $9\cdot\frac{8}{5}$ bu. of potatoes at $62\frac{1}{2}¢$ a bushel.

Problems. — 1. If $\$12\frac{1}{2}$ will pay for 1 mo. work, how much will pay for 8 mo. work?

2. A man who earns \$100 a month spends $\frac{3}{8}$ of what he earns. When he has spent $5\cdot\$37\frac{1}{2}$, how much will he have earned?

3. If $x¢$ is the cost of $37\frac{1}{2}$ ft. of lumber, how many thirds of $x¢$ equal the cost of 100 ft.? of 400 ft.?

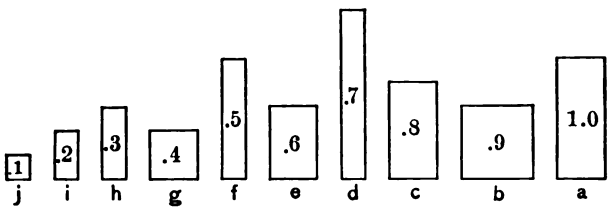
4. At $62\frac{1}{2}¢$ a bushel, how many bushels of wheat can be bought for \$5?

5. The cost of a lot was $\$62\frac{1}{2}$. The ratio of the cost of a house to the cost of the lot was $6\cdot\frac{8}{5}$. What was the cost of the house?

6. $87\frac{1}{2}$ bu. of corn cost $\$x$. How many $\frac{8}{7}$ of $\$x$ equal the cost of 400 bu.?

7. If $87\frac{1}{2}$ acres of land cost \$420, the cost of 400 acres equals how many $\frac{8}{7}$ of \$420? how many dollars?

Draw the units on the blackboard, making j a 4-in. sq.



Ratios. — 1. Discover all the ratios you can. Find the number of times a ratio is repeated.

2. If i is .2, how many tenths in each of the other units?

3. Compare .2 with each of the other units.

4. Write the comparison of .2 with each of the other units.[1]

Ex.: .2 = 2 times .1, $\frac{2}{3}$, $\frac{4}{2}$, $\frac{2}{5}$, $\frac{6}{3}$, $\frac{2}{7}$, $\frac{1}{4}$, $\frac{2}{9}$, $\frac{1}{5}$.

5. Compare .4 with each of the other units. Write the comparison.

6. Compare .6 with each. Write.

7. Compare .8 with each. Write.

8. Compare .9 with each. Write.

9. Compare 1 with each. Write.

[1] Teacher ask the questions which lead to the answers, and write answers on blackboard as they are given. Let the pupils see several comparisons written and read before requiring them to write.

10. Compare 1.0 (ten tenths) with each. Write.

11. What units have the ratio 2? the ratio 3? the ratio 4? the ratio 5?

12. What units have the ratio $\frac{3}{2}$? $\frac{4}{3}$? $\frac{5}{3}$? $\frac{5}{4}$?

13. What units have the ratio $\frac{2}{3}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{4}{5}$?

1. What is the ratio of 1 to .5? 2 is the ratio of what to .5? What is the ratio of 8 to .5? of 7? of 10? of 12?

2. What is the ratio of 1 to .4? $\frac{5}{2}$ is the ratio of what to .4? What is the ratio of 6 to .4?

3. Make sentences like this: $6 \cdot \frac{5}{2}$ is the ratio of 6 to .4?

4. What is the ratio of 1 to .2? of 7 to .2? of 8? of 5? of 9? of 11?

5. If .5 of a ton of coal costs $\$x$, what is the cost of 3 tons? of 12 tons?

6. What is the ratio of 1 to .6? of 5 to .6? of 7? of 6?

7. What is the ratio of 1 lb. to 8 lb.? of 6 lb. to 8 lb.? of 4 lb.? of 3 lb.?

8. What is the ratio of the cost of 1 lb. to the cost of .8 lb.? of 6 lb. to .8 lb.?

9. Make sentences like this: $6 \cdot \frac{5}{4}$ is the ratio of 6 lb. to .8 lb. $6 \cdot \frac{5}{4}$ is the ratio of the cost of 6 lb. to the cost of .8 lb.

10. What is the ratio of 1.2 (twelve tenths) to .6? to .4? to .3? to .2? to .5? to .8? to .9? to 1.0? to 2.4? to 1.8? to 1?

11. Ask and answer questions like this: 4 is the ratio of what to .3? $\frac{2}{3}$ is the ratio of what to 1.8?

12. Write 10 sentences like this: $\frac{2}{3}$ is the ratio of 1.2 to 1.8.

Inferences. — If I see the leaves on the tree move, I infer that the wind blows.

1. If you put some money into your pocket, and soon after find the money gone and a hole in your pocket, what may you infer?

2. If the area of the parlor floor is twice as great as that of the dining-room, what may you infer as to the relative amount of carpet required to cover each? If .3 of a lady's money will pay for a carpet for the dining-room, how many tenths of her money will pay for a carpet for the parlor?

3. When peaches and oranges were selling at the same rate, a lady paid 5 times as much for oranges as for peaches. What may you infer as to the ratio of the number of oranges bought to the number of peaches?

4. Before you can infer the ratio of the cost of a bushel of corn to the cost of a peck, what must you know?

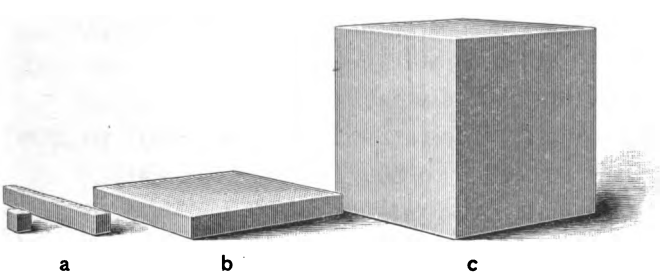
5. If you know that the fare from A to B is 30¢, and from A to C $\frac{1}{3}$ as much, what may you infer as to the relative distance of the two places from A? From knowing the relative distance of the two places from A, what inferences can you make?

6. If you know the number of boys required to sweep 5 rooms in 10 min., what may you infer as to the number required to sweep 1 room in 10 min.? Would this inference be true if the amount of work done in sweeping 1 room were not equal to $\frac{1}{5}$ the amount in sweeping 5 rooms?

7. The ratio of the surface of a table 1 yd. sq. to a table 1 ft. sq. is 9. The amount of paint needed to paint the smaller table equals x. What can you find out from these facts?

Let some pupils give facts and others inferences from facts given.

Make inferences and have pupils seek the basis of the inference, thus: I infer that I can buy 3 times as many bushels of potatoes for \$51 as for \$17. Why?



Make a cube 10″ long of tough check and a prism 10″ by 10″ by 1″ and another 10″ by 1″ by 1″. Place them where they can be observed.

Ratios. — 1. What ratios do you see?

2. What is the ratio of b to a? of b to c? of c to b? of c to a?

3. What part of c equals b? What part of c equals a?

4. b equals how many a's? c equals how many a's?

5. What is the ratio of c to 5 layers of c? to 2 layers? to 4? to 6? to 8?

6. What is the ratio of 5 layers of c to c? of 2 layers? of 4? of 6? of 8? of 7?

7. a equals how many cubes 1″ by 1″ by 1″?

8. Into how many cubes 1″ by 1″ by 1″ can b be cut?

9. c equals how many b's?

10. If we call b 100, c equals how many 100's?

11. What is the ratio of c to 500? to 200? to 400? to 600? to 800?

12. 1000 equals what?

13. What is the ratio of 1000 to 500? to 200? to 400? to 600? to 800? to 700? to 300?

14. What is the ratio of 6000 to 500? to 200? to 400? to 800? to 600?

15. What is the ratio of 1000 to 250? of 5000 to 250? of 7000? of 9000?

1. The ratio of 50 to 100 equals the ratio of what to 1000?

2. The ratio of 25 to 100 equals the ratio of what to 1000?

3. The ratio of 75 to 100 equals the ratio of what to 1000?

4. The ratio of $37\frac{1}{2}$ to 100 equals the ratio of what to 1000?

5. The ratio of 90 to 100 equals the ratio of what to 1000?

6. Make sentences like this: The ratio of 4 to 2 equals the ratio of 6 to 3.

1. What is the least number of equal cubes into which c can be cut?

2. What is the ratio of c to 4 of these cubes? to 2? to 6? to 8? to 1?

3. What is the length of 1 of the 8 equal cubes into which c can be cut?

4. How high a pillar can be built of the 8 equal cubes into which c can be cut?

5. How many layers 5 in. by 5 in. by 1 in. in a 5-in. cube?

6. How many cubic inches in 1 layer? in the 5?

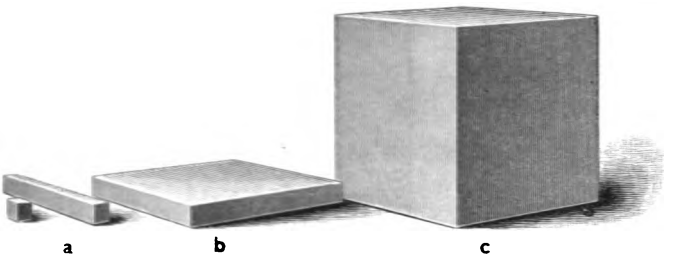
7. What is the ratio of c to 125 cu. in.?

8. What is the ratio of 1000 to 125?

9. The ratio of 1000 to 125 equals the ratio of what to 100?

10. How many square inches in the lateral surface of b? How many rows of 40 sq. in. in the lateral surface of c?

11. How many rows of 10 sq. in. in the sum of the bases of c? of b? of a?



1. b equals what part of c?

2. How many rows equal to a in one layer of c?

3. Into how many cubes 1″ by 1″ by 1″ can a be cut?

4. How many rows of 10 cubes in one layer of c?

5. How many cubes in one layer?

6. What is the ratio of c to a layer? of a layer to a row? of a row to a cube? of a layer to a cube? of c to one of the small cubes?

7. What is the ratio of c to 5 layers? of a layer to 5 rows?

8. Make five sentences like this: 2 c's and 3 layers equal 23 layers.

9. Make five like this: 43 layers equal 4 c's and 3 layers.

10. Make five like this: 3 c's, 7 layers, 4 rows equal 374 rows.

11. Make five like this: 673 rows equal 6 c's, 7 layers, 3 rows.

1. How many inch cubes in 3 c's, 4 layers? in 7 c's, 8 layers? in 36 c's, 5 layers? in 3 c's, 2

rows? in 72 *c*'s, 9 rows? in 3 *c*'s, 3 layers, 3 rows, 3 cubes? in 7 *c*'s, 7 layers, 7 rows, 7 cubes? in 15 *c*'s, 5 rows, 6 cubes?

2. If we call *c* 1 thousand, what shall we call a layer? a row? a cube?

3. How many thousand, hundreds, tens, and ones in 7 *c*'s, 4 layers, 2 rows, 6 cubes?

4. Make five sentences like this: There are 7 thousand, 4 hundred, 2 tens, 6 ones in 7 *c*'s, 4 layers, 2 rows, 6 cubes.

5. Make sentences like this: 4800 cubes equal 4 *c*'s, 8 layers.

6. Make sentences like this: 5285 cubes equal 5 *c*'s, 2 layers, 8 rows, 5 cubes.

7. Tell how many hundreds, tens, and ones in each of the following: 254 sticks; 523 sticks; 806 sticks; 724 boys; 503 boys; 843 lb.; 804 marbles; 307 doz.; 760 knives; 574; 909; 444.

8. Tell how many thousand, hundreds, tens, and ones in each: 8435; 6876; 9042; 7406; 5804; 8695; 43,802; 27,607; 13,004.

Ratios — Imaging. — Learn by copying and by imaging, not by pronouncing.

Under right conditions simple necessary relations are easily perceived, and fixed by use in gaining new ideas. Good teaching gives the pupil a firm basis. It carefully avoids the waste of trying to impress forms or secure dexterity at the expense of growth. It induces through varied

activities the clear, elementary ideas which make possible correct and rapid judgments in more complex work.[1]

6 12 18 24 30 36 42 48 54 60 66 72
1 2 3 4 5 6 7 8 9 10 11 12

6 is the ratio of 6 to 1, of 12 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
6 12 18 24 30 36 42 48 54 60 66 72

$\frac{1}{6}$ is the ratio of 1 to 6, of 2 to 12, etc.

7 14 21 28 35 42 49 56 63 70 77 84
1 2 3 4 5 6 7 8 9 10 11 12

7 is the ratio of 7 to 1, of 14 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
7 14 21 28 35 42 49 56 63 70 77 84

$\frac{1}{7}$ is the ratio of 1 to 7, of 2 to 14, etc.

8 16 24 32 40 48 56 64 72 80 88 96
1 2 3 4 5 6 7 8 9 10 11 12

8 is the ratio of 8 to 1, of 16 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
8 16 24 32 40 48 56 64 72 80 88 96

$\frac{1}{8}$ is the ratio of 1 to 8, of 2 to 16, etc.

[1] "The possibility of all education is based upon the existence of this power . . . of organizing conscious actions into more or less unconscious or reflex operations." — Prof. Huxley.

"The result of the learning process is the conversion of conscious effort into automatic activity, leaving the attention free to seize upon new knowledge and assimilate it with present possessions." — Geo. P. Brown.

9 18 27 36 45 54 63 72 81 90 99 108
1 2 3 4 5 6 7 8 9 10 11 12

9 is the ratio of 9 to 1, of 18 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
9 18 27 36 45 54 63 72 81 90 99 108

$\frac{1}{9}$ is the ratio of 1 to 9, of 2 to 18, etc.

10 20 30 40 50 60 70 80 90 100 110 120
1 2 3 4 5 6 7 8 9 10 11 12

10 is the ratio of 10 to 1, of 20 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
10 20 30 40 50 60 70 80 90 100 110 120

$\frac{1}{10}$ is the ratio of 1 to 10, of 2 to 20, etc.

11 22 33 44 55 66 77 88 99 110 121 132
1 2 3 4 5 6 7 8 9 10 11 12

11 is the ratio of 11 to 1, of 22 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
11 22 33 44 55 66 77 88 99 110 121 132

$\frac{1}{11}$ is the ratio of 1 to 11, of 2 to 22, etc.

12 24 36 48 60 72 84 96 108 120 132 144
1 2 3 4 5 6 7 8 9 10 11 12

12 is the ratio of 12 to 1, of 24 to 2, etc.

1 2 3 4 5 6 7 8 9 10 11 12
12 24 36 48 60 72 84 96 108 120 132 144

$\frac{1}{12}$ is the ratio of 1 to 12, of 2 to 24, etc.

See method of work, pages 74, 75.

30	10	20	20	10
20	10	20	10	30
50	20	40	30	40

Imaging. — 1. Practice imaging the combination and sums, slowly at first.

2. Image combinations and sums and pronounce, thus: 50, 20, 40, 30, 40.

3. Image and pronounce the *number* of tens only, thus: 5, 2, 4, 3, 4.

31	23	12	21	12	13	23
21	22	11	32	32	12	22

Practice reading at sight 52, 45, etc.

Teacher pronounce 31, 21. Pupils arrange mentally as on a slate or blackboard, thus: $\begin{array}{r} 31 \\ 21 \\ \hline 52 \end{array}$ and give the sum.

Practice reading at sight, 254, etc.

122	231	232	321	132
132	211	122	223	112

In the following, practice by the methods suggested above.

40	20	10	50	30
30	40	40	20	30

42	34	12	53	31	34
34	42	44	23	44	31

423	132	214	342
344	444	443	314

80	20	10	30	30
10	60	50	50	60

82	61	18	36	33
16	25	51	53	65

826	258	332	325
162	611	566	663

50	70	10	40	20
40	10	60	40	70

51	16	41	24	47
47	71	46	74	51

516	165	142	754
471	714	647	244

50	60	60	70	50
50	60	40	70	60

53	63	64	73	57
52	62	45	74	62

537	625	623	534
522	634	474	622

70	60	70	80	50
40	70	30	80	70

73	73	34	82	53
42	63	72	86	71

750	631	725	504
440	741	372	745

80	70	80	50	80
40	80	30	80	20

82	83	32	53	24
45	73	85	82	84

821	832	852	825
453	735	326	524

50	90	80	90	90
90	20	60	80	60

53	24	83	83	62
93	94	65	96	96

923	945	614	842
524	243	832	947

90	30	70	90	90
10	90	90	40	90

12	91	93	97	93
97	35	74	42	96

973	343	924	924
123	934	764	453

6	8	2	5	4	7	2	3	9	5	4	8	5
7	3	8	8	7	3	9	9	1	6	7	6	5
8	9	8	8	5	7	7	6	8	7	9	6	8
9	9	4	7	9	5	9	6	9	7	4	9	8

Imaging. — Practice imaging two figures with the sum.

Practice imaging, $\begin{matrix}5\\5\\8\\8\end{matrix}$ thus: $\begin{matrix}10\\16.\\26\end{matrix}$ Do not think 8, 16, 21, 26, but change the four figures into $\begin{matrix}10\\16\\26\end{matrix}$ at sight.

Practice until you can see the sum of any four figures at sight.

Practice adding 1 to the sum.

Ex.: Think 27 instead of 26 as sum of the first column at the right. Practice adding 2, then 3, to the sum.

8	7	9	8	6	5	8
45	18	24	35	28	47	59

In seeing the sum of 59 and 8, change the 59 and 8 to $\begin{matrix}5\\17.\\67\end{matrix}$

Practice imaging until you can give results instantly.

Make other similar tables.

26	37	19	48	67	93	53	27
57	18	46	53	79	47	67	72
49	64	39	39	26	68	85	25
38	27	26	56	87	75	39	59

37	81	94	47	83	27	12	34
46	24	47	38	49	56	35	89
90	57	62	56	42	76	83	78
53	46	88	29	78	85	37	67

69	98	24	49	63	58	94	49
54	34	56	75	77	42	83	38
15	46	57	42	88	66	74	74
83	78	94	65	28	77	39	39

639	312	289	457	754	298	753	357
249	542	361	119	163	928	357	753
732	987	394	475	631	892	837	738
345	731	321	457	163	306	524	425

Problems. — 1. A man bought butter at 20¢ a pound and sold it for a sum equal to $\frac{4}{5}$ of the cost. At what did he sell it?

2. A merchant sells flour for $\frac{1}{5}$ more than its cost. What is the ratio of the selling price to the cost? of the cost to the selling price?

3. If a is the cost of a slate and b the cost of a ball, what is the cost of both?

4.

If the part of the line called m is 7 in. and the part called o 5 in., what is the length of the line? How many more inches in the sum of m and o than in o? than in m?

5. What is the ratio of $20¢$ to $4¢$? What is the ratio of the number of yards of tape which can be bought for $20¢$ to the number which can be bought for $4¢$? What is the ratio of the number which can be bought for $\frac{1}{2}$ of $20¢$ to the number which can be bought for $\frac{1}{2}$ of $4¢$?

6. If the ratio of $\$x$ to $\$y$ is 7, what is the ratio of the number of caps which can be purchased for $\$x$ to the number which can be purchased for $\$y$?

7. If the ratio of $\$x$ to $\$y$ is m, what is the ratio of the number of caps which can be purchased for $\$x$ to the number which can be purchased for $\$y$?

8. If 2 is the ratio of 28 to 14, what is the ratio of the number of bushels of apples that can be bought for $28 to the number that can be bought for $14?

9. How does knowing the ratio of 15 to 20 help you to know at once the part of a yard of ribbon at $20¢$ a yard which you can buy for $15¢$?

Let pupils give other ratios and tell things they see to be true from knowing these ratios.

10. If you know that $\$m$ is the cost of 4 hats, what may you infer as to the cost of 3? Why? The ratio of 3 to 4 equals the ratio of what to $\$m$?

11. If 3 boxes of currants cost 21¢, what is the cost of 5 boxes? The ratio of 5 to 3 equals the ratio of the cost of what to the cost of 3?

12. If you know the cost of 3 oranges and the number of oranges bought, what can you find? The ratio of the number bought to 3 equals the ratio of what?

13. If 5 is the ratio of the entire amount paid for sugar to the amount paid for 3 lb., how many pounds are bought?

14. $21 will support a family how many times as long as $7?

15. If $\$a$ equal 11 times $\$b$, $\$a$ will buy how many times as much as $\$b$?

16. The ratio of one line to another equals the ratio of a yard to a foot. $2\frac{2}{3}$ yd. is the length of the shorter line. What is the length of the longer?

Place bundles of hundreds, tens, and ones where they can be handled and compared.

Ratios. — 1. If we call one of the largest bundles 1, what shall we call one of the units next in size to the largest? What shall we call one of the smallest units?

2. Show me a 1. Show me .1, .3, .2, .5. Show me .01, .07, .02, .11.

3. What is the ratio of 1 to .1? of .1 to .01? of 1 to .01?

4. What is the ratio of .01 to .1? .1 equals how many hundredths?

5. Compare 1 with each of the others. Compare .1 with each. Compare .01 with each.

Pupils practice selecting units and telling what they have.

Ex.: Pupil selects 3 ones, .2, and .04, unites them, and says: "This unit is the sum of 3, .2, .04.

Pupils select, tell what they have, and teacher write on the blackboard.

Ex.: Pupil selects 2, .3, .05 and says: "This unit is the sum of 2, .3, .05 and equals 2 and .35, or 235 hundredths." Teacher writes 2.35.

Teacher write names of the units on the blackboard and pupils find them.

Ex.: Teacher writes 3.04 and pupil finds the 3 and the .04.

Pupils write on the blackboard and tell what they have written.

Ex.: Pupil writes 27.14 and reads 27 and .14.

Teacher write name of a unit on the blackboard. Pupil find part of unit asked for.

Ex.: Teacher writes 23.42 and says: "Find the part of the unit represented by the second figure

from the right. Show me the part of the unit represented by the first figure."

Teacher or pupil name units and pupils write.

Ex.: Teacher or pupil says "12.05," and pupils write. Practice.

Select units of ones and tenths and tell the number of tenths.

Ex.: Pupil selects 3 ones and 4 tenths and says: "This is 3.4 ($\frac{3}{1}\frac{4}{0}$)."

Select units and tell how many hundredths.

Ex.: Pupil selects 2.3 and says: "2.3 ($\frac{2}{1}\frac{3}{0}$) equals 2.30 ($\frac{2}{1}\frac{3}{0}\frac{0}{0}$)."

Place silver dollars, dimes, and pennies where they can be observed and handled.

1. A dime equals how many hundredths of a dollar? One dollar equals how many tenths of a dollar? how many hundredths? 3 dimes and 4 pennies equal how many hundredths of $1?

2. If we call the silver dollar 1, what ought we to call a dime? a penny? What is the ratio of the value of $1 to the value of a dime? to a penny? of a dime to a dollar? to a penny? of a penny to a dime? to a dollar?

3. Pupils practice selecting units and telling what they have.

Ex.: Pupil selects $2, 3 dimes, and 4 pennies and says: "This unit is the sum of $2, 3 dimes, and 4 pennies and equals 2 and $\frac{3}{1}\frac{4}{0}\frac{}{0}$ dollars."

4. Pupils select, tell what they have, and teacher write on the blackboard.

Ex.: Pupil selects \$3, 4 dimes, and 2 pennies and says: "3 and $\frac{42}{100}$ dollars," and teacher writes \$3.42.

Teacher write names and read, and pupils find the units.

Ex.: Teacher writes \$2.34, and a pupil selects unit.

Read each of the following in two ways:

Ex.: 8 and $\frac{64}{100}$ dollars. \$8 and 64¢.

\$7.56	\$6.32	\$9.21	\$7.60
.27	7.15	18.10	11.03
25.36	4.05	15.16	13.50

1. Carrie paid \$.85 for a doll, \$1.15 for slippers, and \$.10 for postage. How much for all?

2. Lucy bought 2 yd. of ribbon at \$.12$\frac{1}{2}$ per yard, 1 yd. of cashmere for \$.87$\frac{1}{2}$, and 1 yd. of flannel for \$.37$\frac{1}{2}$. What was the amount of her bill?

3. Fred has \$2.03 in his bank. How much must he add to this that he may have \$5.10?

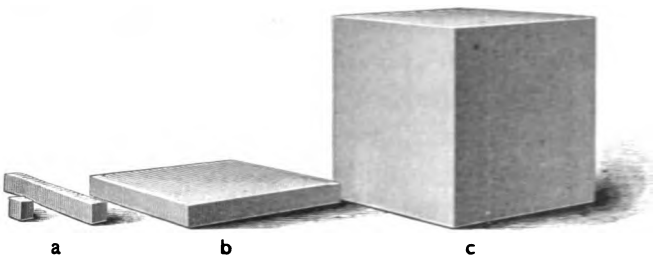
4. In a sack there are 3 pk. and 2 qt. of seed-corn. How much must be added to this that there may be a bushel?

5. Fred has 8 dimes and Walter \$.50. What is the ratio of Walter's money to Fred's?

6. What is the ratio of \$.60 to \$.15? of 3 · \$.60 to 3 · \$.15?

7. For 3 · \$.60 a grocer can buy how many packages of oatmeal at \$.15 a package?

8. A boy saves \$.25 a week. In how many weeks does he save \$5? How much does he save in a year?



Place a 5-in. cube and a prism 5 in. by 5 in. by $\frac{1}{2}$ in. and a prism $\frac{1}{2}$ in. by $\frac{1}{2}$ in. by 5 in. where they can be observed.

1. b equals what part of c? a equals what part of .1 of c? If b is a layer, c equals how many layers?

2. Write ten sentences like this: 2 layers, 3 rows equal .23 of c (read, 23 hundredths of c).

3. Write 10 sentences like this: .86 of c equals 8 layers, 6 rows.

4. Tell what each of the following represents in layers and rows: .25, .18, .20, .2, .29, .09, .46, .93, .02, .3, .30.

5. The unit c equals how many hundredths of another c? 2 c's, 6 layers, and 5 rows of another c equal how many hundredths of c? *Ans.:* 2.65 (read, $\frac{2}{1}\frac{6}{0}\frac{5}{0}$). If I think of 7 c's, 4 layers, 9 rows of another c, I can write it thus: 7.49, (read, $\frac{7}{1}\frac{4}{0}\frac{9}{0}$, or $\frac{7}{1}\frac{4}{0}$ and $\frac{9}{1\,0\,0}$, or 7 and $\frac{4}{1\,0}$ and $\frac{9}{1\,0\,0}$, or 7 and $\frac{4\,9}{1\,0\,0}$).

6. Make sentences like this: 4 c's, 3 layers, 7 rows equal 4.37. Read each in different ways. Show the units that make the sum.

7. Make sticks into bundles of 10's. Make bundles of 10·10's, or of 100's. Make one bundle of 10·100's, or 1000.

8. If the largest bundle is 1, the next in size is what? one of the smallest is what?

9. Show me the 1. Show me a .1. Show me .01. Show .25, .21, .02, .32, .19, 1.11 ($1\frac{11}{100}$), 1.02, 1.22, 1.3, 1.30, 1.20, 1.02.

1. What is the ratio of the larger to the smaller unit? of the smaller to the larger?

50	64	85	720	805	5
5	6.4	8.5	72	80.5	.5

2. Write .1 of each of the following: 2; 7; 85; 760; 80; 2.7; 41.

3. Pupils name units and express .1 of the units named.

4. Practice reading .1 of different units.

Ex.: \$94.7 is .1 of \$947.

5. What is .3 of 65? What is .1 of 65? 6.5 = .1 of 65. What, then, equals .3 of 65? 3·6.5 = .3 of 65.

6. What is 1.3 ($1\frac{3}{10}$) of 65?

7. What is 2.3 ($2\frac{3}{10}$) of 547 bu.? What is .1 of 547 bu.? What, then, is 2.3?

1. What is the ratio of the larger to the smaller unit? of the smaller to the larger?

800	7200	64	8465	4
8	72	.64	84.65	.04

2. Write .01 of each of the following: $200; 80 yds. 27 oz. 164; $5; 8.4.

3. Make many sentences like this: .01 of $843 is $8.43.

4. What is .01 of $200? of 80 yd.? of 27 oz.? of 164? of 15? of $5? of 8.4?

5. What is .05 of 874 ft.?

8.74 What is .01 of 874 ft.?
5 What, then, equals .05 of 874 ft.?
43.70

6. What is .22 of 874 ft.?

7. What is 3.22 ($\frac{322}{100}$) of 874 ft.?

8. Make many sentences like this: .07 of 342 oz. equals 7 3.42 oz.

Equations — Sums. —

1.	2.	3.	4.	5.
3.37	25.4	$5.31	50.7	1.92
7.83	69.8	1.35	28.9	5.09
8.37	89.6	7.35	62.8	3.87
9.10	74.1	7.36	54.7	2.46

6.	7.	8.	9.
70183	8234	31795	42523
91456	69300	43797	305236
5788	35852	24900	561352
49473	15279	34651	632536

10.	11.	12.	13.
2034	8988	7977	8583
8060	5421	6503	6430
5032	6167	5456	5082
8797	5878	6432	6453
4632	8976	5873	8793

14.	15.	16.	17.
1234	$950.06	$38.	$796.08
4123	40.24	7.40	.93
6421	307.18	580.	600.02
6862	490.20	14.10	8.10

18.	19.	20.	21.
840.	4.7	100.05	31.08
243.07	400.875	.50	7.316
88.431	109.98	76.010	1.625
946.25	3.17	307.	18.07

22.	23.	24.	25.
$842	375 dy.	247 oz.	846 ft.
$976	846 dy.	200 oz.	747 ft.
$874	303 dy.	998 oz.	584 ft.
$346	974 dy.	745 oz.	206 ft.

26. What is the sum of $675, $843, $604, and $64?

27. What is the sum of 856 bu., 655 bu., 27 bu., and 803 bu.?

28. What is the sum of 304 yd., 67 yd., 902 yd., and 45 yd.?

29. How many acres in 4 farms, each containing 160 acres?

30. A man had 245 sheep in each of two fields and 673 in another. How many sheep did he have?

1. What is the amount of a lady's bill for the following?

1 bbl. flour,	$ 6.00
1 bbl. sugar,	10.50
1 broom,	.30
1 box soap,	3.60

2. If a ship sailed 212 mi. one day, 225 the next, 240 the next, and 257 the next, how far did it sail in the 4 dy.?

3. How much milk does a dealer sell in 3 dy. who sells as follows for the first, second, and third days?

gal.	qt.	pt.
10	1	0
15	1	1
22	1	1

4. What is the sum of the following?

bu.	pk.	qt.	yd.	ft.	in.
12	2	3	5	2	7
15	1	4	11	1	5
16	1	1	8	2	6

5. In a city there are 5 wards. The first has a population of 4845; the second, 2750; the third, 4345; the fourth, 6000; the fifth, 7012. What is the population of the city?

6. A man paid $500 for a lot. He built a house which cost $756; for a sidewalk he paid $12.50; for a barn $146; and for other improvements, $75.07. What did the place cost him?

7. If a horse travels 27.6 mi. one day, 24.5 another day, and 30.25 another, how far does it travel in the 3 dy.?

8. In one package there are 5 lb. 3 oz. of tea, in another 2 lb. 4 oz., and in another 4 lb. 3 oz. What is the weight of the tea?

a

3	4	2	5	6	7	10	8
1	1	1	1	1	1	1	1

Equations — Differences. — 1. Think the number which added to the lower will make the sum equal to the upper. Do not pronounce, but think or picture 2 in the first combination.

2. Copy and write the number which added to the lower will make the sum equal to the upper.

3. Practice in the same manner, from day to day, on the following tables:[1]

[1] "Fatigue in any form is fatal to memory: the received impressions are not fixed; reproduction is slow, often impossible." — Herbert Spencer.

b

3	5	4	7	9	6	8	11	10
2	2	2	2	2	2	2	2	2

c

7	6	5	9	11	12	10	8	4
3	3	3	3	3	3	3	3	3

d

8	7	5	6	9	11	10	12	13
4	4	4	4	4	4	4	4	4

e

7	9	8	11	12	10	13	15	14
6	6	6	6	6	6	6	6	6

f

12	11	9	8	10	13	16	14	15
7	7	7	7	7	7	7	7	7

g

16	15	10	11	14	12	17	9	13
8	8	8	8	8	8	8	8	8

h

15	17	16	18	14	13	12	11	10
9	9	9	9	9	9	9	9	9

i

11	15	17	16	18	14	13	17	11
7	9	8	7	9	5	4	9	4

Equations — Differences. — 1. What must be added to b to make the sum equal to a?

(a) 546 4 and what equal 6?
(b) 334 3 tens and what equal 4 tens?
(c) 212 3 hundred and what equal 5 hundred?

What then must be added to b to make the sum equal to a?

How much larger is a than b?
How much larger is a than c?
How much less is b than a?
How much less is c than a?

2. Henry has 746 sticks and James 124 sticks. How many more has Henry than James?

746 4 and what equal 6?
124 2 tens and what equal 4 tens?
622 1 hundred and what equal 7 hundred?

There are four equations in this problem. What are they?

Henry has how many more sticks than James?

How many less has James than Henry?

What is the difference between 746 sticks and 124 sticks?

If Henry gives away 124, how many will he have left? if he gives away 622?

3. In each of the following, what must be added to the smaller that it may equal the larger?

Image the equations.

493	479	687	798	794	985	688	795	335
182	373	225	586	372	572	464	682	124
778	594	758	983	593	953	886	854	750
435	314	245	471	281	622	543	503	500

4. Frank earns $4.50 a week and spends $3.25. How much more does he earn than spend? He spends how much less than he earns?

What is the difference between the money earned and the money spent?

5. In one piece of carpet there are 125.4 yd. and in another 146.7 yd. What is the difference in the number of yards in the two pieces?

From the larger piece there can be cut how many yards more than from the smaller piece?

If the difference between the two pieces were added to the smaller the sum would equal what?

6. A man buys a farm for $8794. At the time of purchase he pays $6572. How much does he owe on the farm?

7. In one bin there are 687 bu. 3 pk. 6 qt. of wheat, in another 547 bu. 2 pk. 4 qt. How much must be added to the wheat in the smaller bin that it may equal the amount in the larger one?

8. A lady's purchase amounted to $15.93. She had only $14.32. How much more did she need to pay her bill?

9. In a can there are 5 gal. 3 qt. of oil. If 1 gal. 2 qt. leak out, how much will be left in the can?

How much more in the can before than after leakage?

How much less after than before leakage?

What is the difference between 5 gal. 3 qt. and 1 gal. 2 qt.?

What must be added to 1 gal. 2 qt. that the sum may equal 5 gal. 3 qt.? $5\frac{3}{4}$ gal. less $1\frac{1}{2}$ gal. equal what?

10. For wood sold to a merchant a farmer received groceries worth \$12.50 and \$4.25 in money. What was the value of the wood?

How much more was the wood worth than the groceries?

11. The difference between the length of two lines is 2 ft. 5 in. What must be added to the shorter line that it may equal the longer?

12. Walter has \$1.25, which is \$.50 less than Charles has. How can you find how much Charles has?

13. The sum of the height and the length of a blackboard in this room is how much greater or less than the sum of the height and width of the door?

14. $a + b$ equals \$746. $c + d$ equals \$524. $a + b$ is how much greater than $c + d$?

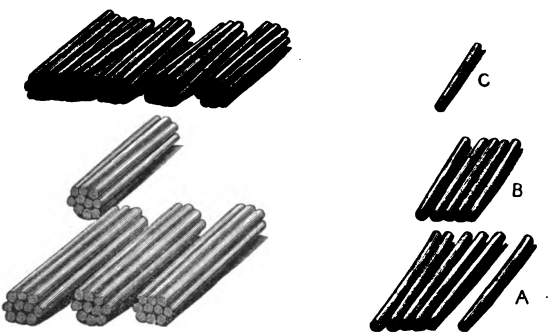
How much must be added to $c + d$ that the amount may equal $a + b$? \$746 − \$122 equals what?

$a + b$ less $c + d$ equals what?

15. $x + y$ equals 200 more than $m + n$. How much must be added to $m + n$ that the amount may equal $x + y$?

$x + y - 200$ equals the sum of what?

$m + n + 200$ equals what?



Equations. — 1. The sum of 36 and 15 equals what?

(*a*) 36 5 and 6 equal what?
(*b*) 15 2 tens and 3 tens equal what?
(*c*) $\overline{51}$ 15 and what equal 51?

(*c*) 51 5 and what equal 11?
(*b*) 15 2 tens and what equal 5 tens?
(*a*) $\overline{36}$ 15 and what equal 51?

51 is how much more than 15?

15 is how much less than 51?

What is the difference between 51 and 15?

If you had 51¢ and should lose 15¢, how much money would you have left?

2. If Mr. B pays $510 for a horse, and Mr. A $150, what is the difference in the cost of the horses?

$510 — 5 tens and what equal 11 tens?
150 — 2 hundred and what equal 5 hundred?
$360 — What, then, is the difference in the cost of the horses?

How much more does B pay than A?

How much less does A pay than B?

A should give his horse and how much in exchange for B's?

In each of the following, find how much greater the larger unit is than the smaller.

Ex.: 825 is how much greater than 276?

825 — 6 and what equal 15? 8 and what
276 — equal 12? 3 and what equal 8? Think
549 — 6, 9, 15; 8, 4, 12; 3, 5, 8.

424	644	584	893	427	986	857	881
115	327	269	368	215	509	819	534
768	848	382	631	166	240	874	736
239	309	279	408	39	29	35	347
292	871	721	833	721	735	542	915
175	255	156	55	67	569	387	576

235 and 465 equal what?

465 — 5 and 5 equal what?
235 — 4 and 6 equal what?
700 — 3 and 4 equal what?

235 and what equal 700?

700 — 5 and what equal 10?
235 — 4 and what equal 10?
465 — 3 and what equal 7?

			yd.	sq. ft.	hr.
$7.40	$49.0	$5.00	58.6	4.50	4.40
2.95	38.9	4.79	28.7	1.33	1.45
.780	7.70	.930	67.0	4.70	.650
.487	5.87	.555	45.6	3.78	.359
$883.4	$73.25	94.53	$6.543	$28.796	5363.9
591.5	34.68	65.75	3.633	4.554	1575.8
5398.2	438.234	234.56	180.020	50.000	2.3845
589.3	249.465	58.97	47.535	38.346	.1578
42.9	567.82	15.87	1.		
1.05	43.315	9.5	.10		

Problems. — 1. Mr. Smith has $243 and Mr. Johnson has $747. How much more money has Mr. Johnson than Mr. Smith?

2. Columbus discovered America in 1492. How many years since?

3. A drover bought 5947 sheep and sold 3240 of them. How many had he left?

4. A lady bought a hat for $5.75 and a shawl for $10.87. How much more did she pay for the shawl than for the hat?

5. For work, machinery, and other expenses a farmer paid in one year $700. Owing to bad weather the crop was worth only $368. What was the loss?

6. A man deposited in a bank at one time $342 and at another $743. He drew out in all $959. How much had he left in the bank?

7. Bought a coat for $18.50, a vest for $3.87, and paid on both $15.25. How much is yet to be paid?

8. A man bought a horse for $325, a harness for $37.25, and a buggy for $250.50. He paid $327. How much is yet to be paid?

9. Find the difference between:

lb.	oz.
124	12
87	14

Write thus:

lb.	oz.
123	28
87	14

10.

yd.	ft.	in.
127	1	10
98	2	8

11.

hr.	min.	sec.
227	20	20
135	15	35

12.

bu.	pk.
8.36	
	3.75

13. What is the difference between 1.875 and .375?

If to .375, 1.5 is added, what is the sum?

1.875 is how much more than 1.5?

1.875 less 1.5 equals what?

14. The sum of x and y equals .625; if y is .250, what is x?

Pupils ask questions on the problems.

15. .625 is how much more than the sum of .250 and .125?

16. 1020 less the sum of 605 and 210 equals what?

17. 716 gal. 2 qt. 1 pt. are how much more than 502 gal. 3 qt. 2 pt.?

18. Tell the difference in yards, feet, and parts of a foot in the length and width of your schoolroom.

19. 27 gal. 2 qt. 1 pt. 3 gi. less 16 gal. 3 qt. 2 pt. 2 gi. equals what?

20. A lady bought goods to the amount of $70.28. She gave the clerk a $100 bill. How much change ought she to receive?

21. If you have the difference between two units and the smaller given, how do you find the larger? Give examples.

Imaging. — Adding is an easy operation if the mind is trained to make distinct pictures of the figures and to arrange them mentally in their proper relation.

Ability to pass freely from one mental representation to another will culminate in rapidity and accuracy in the mechanical work. Flexibility is a condition of mental growth and of ready expressing.

If in thinking the sum of $\begin{matrix}19\\\hline\end{matrix}$ you visualize 18, 2, arranged mentally, thus: $\begin{matrix}2\\\hline 18,\end{matrix}$ it is easy to see 38. Try it.

If in thinking the sum of $\begin{matrix}27\\\hline\end{matrix}$ you visualize 21, 6, arranged mentally, thus: $\begin{matrix}6\\\hline 21,\end{matrix}$ it is easy to see 81. Try it.

19	17	18	16	15	2
19	17	18	16	15	10
					30

Write the figures that must be seen passing mentally from the equal units to their sum.

Ex.: Observe $\begin{matrix}19\\\hline\end{matrix}$ and write $\begin{matrix}2\\\hline 18.\end{matrix}$

Observe $\begin{matrix}19\\\hline 19.\end{matrix}$ Image $\begin{matrix}2\\\hline 18.\\\hline 38\end{matrix}$

25	26	28	27	29
25	26	28	27	29

Observe, image, and write.

Ex.: Observe $\begin{matrix}29\\\hline 29.\end{matrix}$ Image $\begin{matrix}4\\\hline 18.\\\hline 58\end{matrix}$ Write $\begin{matrix}4\\\hline 18.\\\hline 58\end{matrix}$

After you can write the symbols readily, practice making the mental images until you can see the sums instantly.

Master the following tables by the method suggested above:

14	16	15	17	19	18	3
14	16	15	17	19	18	= 24
14	16	15	17	19	18	54
24	26	25	29	27	28	6
24	26	25	29	27	28	= 24
24	26	25	29	27	28	84

Practice thinking the squares of small numbers.

See mentally,

$$\begin{array}{r} 13 \\ \underline{13} \\ 39 \\ \underline{13} \\ 169 \end{array}$$

What is 12^2? 14^2? 22^2? 23^2? 33^2? 32^2? 21^2?

Ratios. — Express the ratio of 24 to 18 in as many different ways as you can.

1 equals what part of 18?

24 equals how many eighteenths of 18?

If 1 is the measuring unit, what expresses the ratio of 24 to 18?

2 equals what part of 18?

24 equals how many ninths of 18?

If 2 is the measuring unit, what expresses the ratio of 24 to 18?

3 equals what part of 18?

24 equals how many sixths of 18?

If 3 is the measuring unit, what expresses the ratio of 24 to 18?

6 equals how many thirds of 18? 24 equals how many thirds of 18?

If 6 is the measuring unit, what expresses the ratio of 24 to 18? of 18 to 24?

In the following, express the ratio of a to b, and of b to a, in different ways.

a	b	a	b
8	6	9 ft.	72 ft.
\$15	\$20	24	12
$1\frac{1}{2}$ bu.	2 bu.	5 ft. 3 in.	7 in.
5 yd.	$2\frac{1}{2}$ yd.	\$129	\$3
1 yd.	$2\frac{1}{2}$ ft.	.3	2.7
1 lb.	14 oz.	.2	2
$\frac{3}{4}$ lb.	$\frac{1}{2}$ lb.	15	5
\$64	\$4	32	36
48	72	21	42
$\frac{4}{9}$	$\frac{6}{9}$	$\frac{4}{6}$	$\frac{10}{12}$

Review the above until you can give the ratios readily.

What is the ratio of 966 to 3?

322 What is the ratio of 9 to 3? of 6?
3) 966 of 966?

Write many easy problems on the blackboard thus:
4) 1248

Pupils give answers at sight, thus: 312 is the ratio of 1248 to 4.

What is the ratio of 968 to 4?

242 What is the ratio of 8 to 4? of 16 to
4) 968 4? of 8 to 4? of 968 to 4?

What is the ratio of the lumber you can buy for $968 to the lumber you can buy for $4?

If you can buy x yd. of carpet for $4, how many x yd. can you buy for $968?

If 3 bu. of wheat cost $4, how many 3 bu. can be bought for $968?

What is the ratio of $968 to $4?

What, then, is the ratio of the wheat that can be bought for $968 to that which can be bought for $4? How much wheat can be bought for $4?

How much, then, can be bought for $968?

What is the ratio of 707 to 5?

$141\frac{2}{5}$ What is the ratio of 5 to 5? of 20 to
5) 707 5? of 5 to 5? of 2 to 5? of 707 to 5?

What is the ratio of 702 to 5?[1]

$140\frac{2}{5}$ What is the ratio of 5 to 5? of 20
5) 702 to 5? of 2 to 5? of 702 to 5?

[1] Keep the pupils thinking the relations, and, without explanation, place the expression of the relations on the blackboard.

In the following, express the ratio of a to b, and of b to a, in the simplest form:

a	b	a	b	a	b
48	4	968	4	1.08	.03
654	6	36	24	915	5
90	15	534	8	124	4
3045	7	75	25	8521	9
840 ft.	5 in.	4872	4	3936	4
4875	5	3234	3	729 pk.	9 pk.
1362	3	7218	9	6765	5
81.72	.09	8562	3	73,265	5
5256	6	9184	4	132.8	.8
3836	7	2384	8	3412	6
7398	9	9352	7	1704	3
$11\frac{2}{3}$	$\frac{7}{3}$	$17\frac{1}{2}$	$2\frac{1}{2}$	31.544	.008

Problems. — 1. If 5 bu. of corn can be bought for \$2, how many bushels can be bought for \$240?

The ratio of \$240 to \$2 equals the ratio of what to 5 bu.?

2. At \$9 a ton, how many tons of hay can be bought for \$8127?

3. At 6 mi. an hour, how long will it take to travel 252 mi.?

What is the ratio of 252 to 6?

The ratio of 252 mi. to 6 mi. equals the ratio of what to 1 hr.?

4. At \$6 a week, in what time can I earn \$500?

5. 1722 acres of land yield how many times as much corn as 7 acres?

6. What is the ratio of $3045 to $7?

$3045 will buy how many times as much as $7?

7. At $10 a barrel, how many barrels of beef can be bought for $1750?

8. How many cloaks at $9 each can be bought for $981?

9. How many 9 sq. ft. are there in 148 sq. ft.? How many square yards? What is the ratio of 148 sq. ft. to a square yard.

10. At $8 a foot, how many feet of land can be bought for $729?

11. At $.08 a yard, how many yards of ribbon can be bought for $.85?

12. At $9 a ton, how many tons of hay can be bought for $3234?

13. $.98 will buy how many times as much as $.08? What is the ratio of $.98 to $.08?

At $.08 a yard, how many yards of gingham can be bought for $.98?

14. 1704 mi. equal how many 3 mi.?

3 mi. equal 1 league. 1704 mi. equal how many leagues?

15. What is the ratio of a league to a mile? of 5 leagues to 5 mi.? of $\frac{1}{4}$ league to $\frac{1}{4}$ mi.?

16. When a ship is 1820 mi. from shore, how many leagues has it sailed?

17. At $.05 a quart, how many pecks of beans can be bought for $1.75?

18. How many quarts of berries at $.10 a quart should be given for 2 pk. 1 qt. of nuts at $.05 a quart?

19. At $.15 each, how many books can be bought for $15?

20. At $.60 a pound, what part of a pound can you buy for $.06?

21. At $1½ a ream, how many reams of paper can you buy for $19½?

22. At $.15 a quart, how many pecks of cherries can be bought for $2.55?

Ratios. — 1. What is the ratio of 2562 to 12?

$$
\begin{array}{r}
213\frac{1}{2} \\
12\overline{)2562} \\
24 \\
\hline
16 \\
12 \\
\hline
42 \\
36 \\
\hline
6
\end{array}
$$

What is the ratio of 24 to 12? of 12 to 12? of 36 to 12? of 6 to 12? of 2562 to 12?

What is the sum of 6, 36, 120, and 2400?
What is the ratio of 24 to 12? of 2400?
What is the ratio of 12 to 12? of 12 tens to 12?
What is the ratio of 36 to 12? of 6?
What is the sum of 200, 10, 3, and ½?
What, then, is the ratio of 2562 to 12?

2. What is the ratio of 11,460 to 45?

$$\begin{array}{r} 254\frac{2}{3} \\ 45)\overline{11460} \\ \underline{90} \\ 246 \\ \underline{225} \\ 210 \\ \underline{180} \\ 30 \end{array}$$

The ratio of 11 to 4 is less than 3; the ratio of 114 to 45 is less than 3.

The ratio of 24 to 4 is 6, but the ratio of 246 to 45 is less than 6.

The ratio of 21 to 4 is a little more than 5, but the ratio of 210 to 45 is less than 5.

$\frac{2}{3}$ is the ratio of 30 to 45.

What is the sum of 30, 180, 2250, and 9000?

What is the ratio of 90 to 45? What, then, of 90 hundred to 45? of 225 to 45? of 225 tens? of 180? of 30?

What is the sum of 200, 50, 4, and $\frac{2}{3}$?

What, then, is the ratio of 11,460 to 45?

In each of the following find the ratio of a to b.

a	b	a	b
1386	21	3650	74
2310	42	1620	27
6840	15	2660	35
1728	12	2340	42
2747	41	4030	54
2380	34	1750	25
1770	40	11,216	16
4250	32	18,300	24
6394	78	35,100	48

Review until the work can be done easily.

Problems. — 1. How many acres will produce 6510 bu. of wheat at 30 bu. to an acre?

The ratio of 6510 to 30 equals the ratio of what to 1?

2. How many 3 acres will produce 11,150 bu. at 75 bu. to 3 acres?

The ratio of 11,150 to 75 equals the ratio of what to 3?

3. There are 27 cu. ft. in a cubic yard. How many cubic yards in 5535 cu. ft.?

4. At the rate of $40 a month, in what time will a man save $780?

5. How many hours are there in 2770 min.?

6. At $.25 each, how many meals can you buy for $5.25?

7. At $.72 a yard, how many yards of cloth can be bought for $60.28?

8. At $.08 a quart, how many quarts of berries can be bought for $4.04?

9. At $.46 a bushel, how many bushels of corn can be bought for $124.43?

10. At 40 mi. an hour, in what time does a train run 3005 mi.?

Ratios. — 1. What is the ratio of 6 cars of coal to 2 cars?

What is the ratio of the weight of 6 cars of coal to the weight of 2?

What is the ratio of the time that 6 cars of coal will last to the time that 2 will last?

What is the ratio of the cost of 6 cars of coal to the cost of 2?

2. If 2 cars of coal cost $68, what is the cost of 6 cars?

$\frac{\overset{3}{\cancel{6}}\ \$68}{\cancel{2}} = \$204$. What is the ratio of 6 to 2?[1]

What, then, is the ratio of the cost of 6 cars to the cost of 2?

What is the cost of 2 cars?

What, then, is the cost of 6 cars?

3 is the ratio of what to $68?

$68	
68	$68
68	3
$204	$204

What is the ratio of $204 to $68? What is the ratio of $68 to $204?

The amount of coal that $204 will buy is how many times as great as the amount $68 will buy?

3. What is the ratio of 20 gal. of oil to 5 gal.?

What, then, is the ratio of the cost of 20 gal. to the cost of 5 gal.? of the time 20 gal. will last to the time 5 gal. will last? of the heat pro-

[1] Previous work should have familiarized the pupil with the relations of magnitude which give significance to 6 and 2. (See p. 23, Teachers' Book.)

duced in burning 20 gal. to the heat produced in burning 5 gal.? of the light made by burning 20 gal. to the light made by burning 5 gal.? of the cost of $\frac{20}{2}$ gal. to the cost of $\frac{5}{2}$ gal.? of 3·20 gal. to 3·5 gal.?

4. If 5 gal. of oil cost $.28, what is the cost of 20 gal.?

What is the ratio of $1.12 to $.28? of $.28 to $1.12?

$1.12 will buy how many times as much oil as $.28?

The ratio of 20 to 5 equals the ratio of what to $.28?

5. From the ratios of the other units compared in problem 1, make problems similar to problem 2.

6. If 4 carriages cost $376, what is the cost of 12?

The ratio of 12 to 4 equals the ratio of what to $376?

7. If 5 acres cost $275, what is the cost of 15 acres.

8. If 8 yards of ribbon cost $2.40, what is the cost of 32 yards?

9. Class write and solve 10 problems similar to problem 4. Teacher make other problems from given problems and result. See problems 4 and 5.

10. Review and have answers given as quickly as possible.

1. 3 is the ratio of what to 385?

385

$\frac{3}{1155}$ 3 is the ratio of 1155 to 385.

Find the other term in each of the following:

2. 6·674	8. 6·805
3. 5·924	9. 6·75.9
4. 7·755	10. 9·8.24
5. 8·982	11. 7·6.78
6. 4·565	12. 9·99.9
7. 9·490	13. 9·37804
14. 5·37804	19. 4·209070
15. 6·47095	20. 9·707080
16. 7·84208	21. 7·4500.96
17. 3·943076	22. 8·659.703
18. 5·384627	23. 6·909.074

Problems. — 1. At $3.85 each, what is the cost of 3 chairs?

What is the ratio of the cost of 3 to the cost of 1?

2. The ratio of $3844 to $674 is 6. What is the ratio of the amount of work for which $3844 will pay to the amount for which $674 will pay?

3. The ratio of 924 to 4620 is $\frac{1}{5}$. What is the ratio of 4620 to 924?

4. At $9.82 per thousand, what will 8000 ft. of lumber cost?

5. 490 sq. yd. equal how many square feet?

6. 37,804 bu. equal how many pecks?

7. What is the cost of 2 yd. 2 ft. of picture molding at 60¢ a yard?

8. At 4¢ a square foot, what is the cost of a crumb cloth 3 yd. long and 2 yd. wide?

9. At 28¢ a pound, what is the cost of 9 lb. 8 oz. of butter?

10. At 30¢ a hundredweight, what is the cost of 8 hundredweight of coal?

11. There are 20 hundredweight in 1 ton. How many hundredweight in $7\frac{1}{2}$ tons?

12. At $14 a ton, what will 30 hundredweight of hay cost?

13. What is the cost of 2 bu. of nuts at 8¢ a quart?

14. There are 24 sheets of paper in a quire and 20 quires in a ream. How many sheets in a ream? in 3 reams?

15. At $.30 a quire, what is the cost of a ream of paper? of $1\frac{1}{2}$ reams?

16. What is the cost of 2 pk. 1 qt. of cherries at 15¢ a quart?

17. A stationer sold 5 quires of paper at 2¢ a sheet. For how much did he sell it?

18. There are 196 lb. in a barrel of flour. What is the cost of a barrel of flour at 2¢ a pound? at $2\frac{1}{2}$¢?

Ratios. — 1. Show units represented by 1 and by .1.

What is the ratio of 1 to .1? of .1 to 1?

2. Show me units represented by 5 and .5.

What is the ratio of 5 to .5? of .5 to 5?

3. Write .1 of 50; of 5; of 12; of .4.

4. What is the ratio of 750 to 75? of 75 to 7.5? Express 10 times 7. 10 times 7.5. 10 times 75. 750 is 10 times as much as what?

5. Express 10 times as much as each of the following: 28, $218, 65 sq. in., 2 lb., 193 tons, $67.45, 9.5 oz., 5.4, .06, 678, 5869, 586.9.

6. What is the ratio of 7500 to 75? of 75 to .75? 75 is 100 times as much as what?

7. Write 100 times as much as each of the following: 24, 5, $65, .8, 72, 748, .94, 6.7, 98.65, .05.

8. Find the other term in each of the following:

40 · 95	60 · 75	700 · 56	300 · 95	800 · 409
20 · 95	20 · 947	600 · 580	400 · 87	900 · $582
40 · 32	50 · 450	580 · 800	409 · 700	300 · 525

9. 35 is the ratio of what to 437?

437 — 5 is the ratio of what to 437?
35
2185 — 30 is the ratio of what to 437?
1311
15295

What is the ratio of 2185 to 437? of 13,110 to 437? of 15,295 to 437?

35 is the ratio of what unit to 437?

$15,295 will buy how many times as much flour as $437?

Find the other term in each of the following, and think the ratio of the term found to the given term, and conversely.

1. 23·472	6. 22·6804	11. 384·3521
2. 14·233	7. 13·2345	12. 232·4202
3. 25·642	8. 16·3232	13. 307·2314
4. 33·425	9. 21·6253	14. 204·4243
5. 24·843	10. 25·3322	15. 342·8000

Problems. — 1. What is the cost of 23 city lots at $472 each?

2. What will it cost to build 22 mi. of railroad at $35,210 a mile?

3. At $3.42 per acre, what is the cost of 8000 acres of swamp land?

4. If a man's savings average $700 a year for 33 yr., how much does he save in 33 yr.?

5. What is the cost of 204 acres of land at $42.25 per acre?

6. What will 6804 tons of coal cost at $6.50 a ton?

7. If a equals the cost of a cord of wood and b equals the number of cords bought, what equals the cost of the wood?

8. How much wheat in 8 loads of 42 bu. 3 pk. each? What does a farmer receive if he sells the wheat at \$1.00 a bushel? at $62\frac{1}{2}¢$ a bushel? at $87\frac{1}{2}¢$? at $37\frac{1}{2}¢$?

9. If 4 yd. 18 in. are required for one curtain, how much is required for two?

10. If a bin holds 27 bu. 3 pk. of potatoes, how much do 7 such bins hold?

11. The ratio of the cost of a farm to the cost of an acre is 25. The cost per acre is \$60. What is the cost of the farm? How many acres in the farm? The cost of 1 acre equals what part of the cost of the farm?

Ratios. — 1. What is the ratio of 2 cars of coal to 6 cars?

What is the ratio of the weight? of the time the coal will last? of the cost? of the ashes it will yield? of the number of tons of coal in each unit?

2. If 6 cars of coal cost \$204, what is the cost of 2 cars?

What is the ratio of the cost of 2 to the cost of 6?

What is the cost of 6 cars? $\frac{1}{3}$ is the ratio of what to \$204?

\$68

~~\$204~~

~~3~~

$\frac{1}{3}$ is the ratio of what to 18 tens? of what to 24? of what to 204?

What is $\frac{1}{3}$ of \$204?

$\frac{1}{3}$ is the ratio of what unit to 204? 3 is the ratio of what to 68?

What is the ratio of 68 to 204? of 204 to 68?

The ratio of 2 to 6 equals the ratio of what to 204?

What is the ratio of 2 to 6? of 68 to 204?

3. $\frac{1}{4}$ is the ratio of what unit to 964?

241

$\frac{964}{4} = 241$. $\frac{1}{4}$ is the ratio of what to 8 hundred? of what to 16 tens? of what to 4?

$\frac{1}{4}$ is the ratio of what unit to 964?

4 is the ratio of what unit to 241?

If 12 weighs 964 lb., what is the weight of 3?

If 3 lasts 241 dy., how many days will 12 last?

If \$964 buys x lb., what part of x lb. will \$241 buy?

4. $\frac{1}{3}$ is the ratio of what to 129?

43

Ex.: $\frac{129}{3} = 43$.

3 is the ratio of what to 43. $\frac{1}{3}$ is the ratio of what to 129? 129 is how many times as large as 43? What part of 129 is as large as 43? What part of 129 equals 2·43's? What is the ratio of 129 to 2·43's? Work with the following in a similar manner:

1.	$\frac{1704}{3}$	$\frac{7936}{8}$	$\frac{2384}{8}$	$\frac{4428}{6}$
2.	$\frac{4974}{6}$	$\frac{9002}{7}$	$\frac{6234}{6}$	$\frac{1485}{5}$
3.	$\frac{8024}{4}$	$\frac{4980}{5}$	$\frac{5008}{8}$	$\frac{4974}{6}$
4.	$\frac{84{\cdot}5}{5}$	$\frac{8{\cdot}48}{4}$	$\frac{5{\cdot}52}{12}$	$\frac{69{\cdot}42}{3}$
5.	$\frac{7{\cdot}74}{6}$	$\frac{879{\cdot}6}{4}$	$\frac{8{\cdot}874}{9}$	$\frac{87{\cdot}92}{7}$

1. Pupils write and solve applied problems using units given above, thus:

If 24 acres of land cost \$1704, what is the cost of 8 acres?

The ratio of 8 to 24 equals the ratio of what to \$1704?

Ask other questions.

2. If \$4974 is divided equally among 30 men, what do 5 men receive?

What ratios are equal?

$\frac{1}{6}$ is the ratio of what to \$4974?

Problems. — 1. If you can buy 10 books for \$6, how many can you buy for \$9?

What is the ratio of 9 to 6?

$\frac{3 \cdot \overset{5}{\cancel{10}}}{\cancel{2}} = 15.$ $\frac{3}{2}$ is the ratio of what to 10?

The ratio of 9 to 6 equals the ratio of what to 10?

2. $\frac{2}{3}$ is the ratio of what to 683?

$\frac{227\frac{2}{3}}{2\cancel{683}\ \cancel{3}} = 455\frac{1}{3}$.

What is $\frac{1}{3}$ of 6 hundred? of 6 tens? of 21? of 2?

What is $\frac{1}{3}$ of 683? What, then, is $\frac{2}{3}$ of 683?

The unit $445\frac{1}{3}$ equals what part of 683?

The unit $445\frac{1}{3}$ is how many times as large as $227\frac{2}{3}$?

What is the ratio of 683 to $227\frac{2}{3}$?

3. If the cost of 52 lb. of butter is \$17.60, what is the cost of 65 lb.?

The ratio of 65 lb. to 52 lb. equals the ratio of what to \$17.60?

$\frac{5 \cdot \$17.60}{4}$ equal what?

4. If 24 pieces of cloth cost \$591, what is the cost of 60 pieces?

5. If a yard of lace costs 35¢, what will $1\frac{1}{2}$ yd. cost?

6. If a yard of ribbon costs 18¢, what will $2\frac{1}{3}$ yd. cost? $2\frac{1}{2}$ yd.? $2\frac{1}{4}$ yd.?

7. If 35 bu. of potatoes can be bought for \$8.75, how much will be required to buy 45 bu.?

8. If a train runs 2160 mi. in 54 hr., how far will it run in 72 hr.?

9. If $\frac{3}{4}$ of the cost of a suit of clothes is \$30, what is the cost of the suit?

Ratios.[1] — 1. What is $\frac{1}{2}$ of 24? of 18? What is the ratio of $\frac{1}{2}$ of 24 to $\frac{1}{2}$ of 18?

2. What is $\frac{1}{3}$ of 24? of 18? What is the ratio of $\frac{1}{3}$ of 24 to $\frac{1}{3}$ of 18?

3. What is $\frac{1}{6}$ of 24? of 18? What is the ratio of $\frac{1}{6}$ of 24 to $\frac{1}{6}$ of 18?

4. What is the simplest expression of the ratio of 24 to 18?

$\frac{4}{3}$ is the ratio of $\frac{1}{6}$ of 24 to $\frac{1}{6}$ of what?

What, then, is the ratio of 24 to 18?

Give examples showing that you can find the ratio of two units by comparing corresponding equal parts.

Find the simplest expression of the ratio of the following units by comparing the smallest equal parts common:

1. What is the ratio of 35 to 25? of $\frac{1}{5}$ of 35 to $\frac{1}{5}$ of 25?

$\frac{35}{25} = \frac{7}{5}$. What is $\frac{1}{5}$ of 35? of 25?
What is the ratio of 7 to 5?
What, then, is the ratio of 35 to 25?

2. What is the ratio of 27 to 36?

$\frac{27}{36} = \frac{3}{4}$. What is $\frac{1}{9}$ of 27? of 36?
What is the ratio of 3 to 4?
What, then, is the ratio of 27 to 36?

3. What is the ratio of 968 to 4?

[1] The ratio of two units is the same as the ratio of their corresponding parts.

$\frac{968}{4} = 242.$

What is $\frac{1}{4}$ of 968? of 4? What is the ratio of $\frac{1}{4}$ of 968 to $\frac{1}{4}$ of 4? What, then, is the ratio of 968 to 4?

What is the ratio of the lumber which can be bought for \$968 to the lumber which can be bought for \$4?

If x yd. of carpet can be bought for \$4, how many x yd. can be bought for \$968?

4. How many acres will produce 6510 bu. of wheat at 30 bu. to an acre?

$\frac{6510}{30} = 217.$

What is $\frac{1}{18}$ of 6510? of 30? What is the ratio of $\frac{1}{18}$ of 6510 to $\frac{1}{18}$ of 30?

What, then, is the ratio of 6510 to 30?

5. At \$15 a month, in what time can a boy earn \$605?

Find the ratio of the following units by comparing equal parts:

a	b	a	b	a	b
15	18	72	24	16	12
48	30	35	40	10	72
16	100	288	160	100	80
60	240	15	25	125	25
18	36	54	144	420	108
420	162	98	21	110	75
63	71	44	66	24	40
60	32	18	63	35	15
55	75	100	44	17	170
12	2562	42	2340	256	1728

a	b	a	b
5 bu. 1 pk.	1 bu. 2 pk.	2 yd. 1 ft.	2 yd. 2 ft.
1 hr. 30 min.	1 hr. 15 min.	8 gal. 3 qt.	5 gal. 1 qt.
$7\frac{1}{2}$	5	$4\frac{2}{3}$	$5\frac{1}{3}$
2.7	1.8	5	.4

1. What is the ratio of 1386 to 21?

$$\frac{1386}{21}$$

At \$21 each, how many cows can be bought for \$1386?

2. If the ratio of $\frac{1}{6}$ of 2310 to $\frac{1}{6}$ of 42 is 55, what is the ratio of 2310 to 42?

3. At \$42 an acre, how much land can you buy for \$2310?

At \$55 an acre, how much land can you buy for \$2310?

4. If the ratio of $\frac{1}{12}$ of 24 to $\frac{1}{12}$ of 18,300 is $762\frac{1}{2}$, what is the ratio of 18,300 to 24?

At \$24 each, how many acres can be bought for \$18,300?

5. If the ratio of $\frac{1}{16}$ of 2048 to $\frac{1}{16}$ of 256 is 8, what is the ratio of 2048 to 256?

6. If a train runs 256 mi. in x hr., in how many x hr. can it run 2048 mi.?

7. If the ratio of 2048 to 256 is 8, what is the ratio of 256 to 2048?

8. If the ratio of $\frac{1}{7}$ of 21 to $\frac{1}{7}$ of 1386 is $\frac{3}{198}$, what is the ratio of 21 to 1386?

9. If the ratio of $\frac{1}{6}$ of 12 to $\frac{1}{6}$ of 2562 is $\frac{2}{427}$, what is the ratio of 12 to 2562?

10. What is the ratio of $\frac{1}{14}$ of 42 to $\frac{1}{14}$ of 1750? of 42 to 1750? of 1750 to 42?

11. What is the ratio of $\frac{1}{5}$ of 2660 to $\frac{1}{5}$ of 35? of 2660 to 35? of 35 to 2660?

12. At \$35 each, how many cows can you buy for \$2660?

13. If \$2660 will pay for a farm, \$35 will pay for what part of it?

14. The ratio of $\frac{1}{5}$ of a man's money to $\frac{1}{5}$ of \$25 is 70. What is the ratio of his money to \$25?

15. What is the ratio of $\frac{1}{9}$ of 5535 to $\frac{1}{9}$ of 27?

There are 27 cu. ft. in a cubic yard. How many cubic yards in 5535 cu. ft.?

16. At \$40 a month, in what time does a man save \$780?

17. At 40 mi. an hour, in what time does a train run 3005 mi.?

18. Among how many men can \$4800 be divided, giving each man \$150?

19. \$3750 paid for a farm at \$25 per acre. How many acres in the farm?

What ratios are equal?

20. A company spent \$60,000 for land, paying \$500 for every 40 acres. How many 40 acres were bought?

What ratios are equal?

21. The ratio of $\frac{1}{12}$ of the cost of a farm to $\frac{1}{12}$ of \$60 is 160. What is the cost of the farm?

22. The ratio of $\frac{1}{3}$ of a to $\frac{1}{3}$ of b is 12. What is the ratio of a to b?

If b is 72, what is a?

At no stage should the pupil be asked to solve problems without the images through which the mind moves to a conclusion. He cannot analyze that which he does not see. Trying to solve a problem without the representative power needed to grasp the problem as a whole, to keep the given relations in view, is futile. It enfeebles the mind. We wrong the child when we exhaust his energy by explanations concerning processes and facts which would be seen at once if the realities were rightly presented to minds fitly prepared.

Problems. — 1. If a man sells 3 horses at \$175 each and 9 cows at \$35 each, how much ought he to receive?

2. Mr. Wells bought 3 acres of land at \$30 an acre and 2 acres at \$75 an acre, and paid for it with city property at \$25 a foot. How many feet did it take?

3. A woman bought 4 yd. of calico at 8¢ a yard, 2 collars at 20¢ each, and a thimble for 40¢. She gave in exchange 3 lb. of butter at 25¢ a pound. How much did she still owe?

4. Mr. Harper teaches 10 mo. at a salary of \$1200. How much is that a month?

5. If a clerk receives \$50 a month, in what time does he earn \$650?

6. At 90¢ for 3 doz. marbles, how many 90¢ will 216 marbles cost?

7. A milliner bought 8 hats for $6 and sold them for $1.25 each. The gain equaled what part of the cost?

8. If 7 bottles of perfumery cost $5.60, what is the cost of 15 bottles?

9. If 13 tons of coal cost $104.65, what is the cost of 52 tons?

10. If 18 acres of land cost $360, what is the cost of 27 acres?

11. If $\frac{4}{9}$ of a barrel of flour cost $5, what does a barrel cost?

12. If $\frac{3}{8}$ of a farm is worth $2700, what is the farm worth?

13. If $\frac{3}{15}$ of a farm cost $3725, how much does $\frac{9}{15}$ of it cost?

14. At the rate of 80 acres for $2400, how many acres can be bought for $7200?

15. A man goes 60 mi. north; he then travels to a town 24 mi. south of his starting-place. How far does he travel?

16. Two men start from the same place and travel in the same direction. One travels 52 mi. a day and the other 38 mi. a day. How for apart are they at the end of 3 days?

17. If they had traveled in opposite directions, how far apart would they have been at the end of 3 days?

18. Two boys had a lemonade stand; one put in 35¢, the other 50¢. What part of the profit should each have?

19. A grocer bought flour at $3.50 a barrel and sold it at $4 a barrel. How many barrels must he sell to make $12?

20. A dealer buys milk at the rate of 4 gal. for 25¢ and sells it at 5¢ a pint. How much does he gain on 12 gal.?

21. A boy paid 12¢ for an orange and 2 apples, the orange costing as much as the 2 apples. What was the cost of an apple?

22. A drover sold a horse and 2 cows for $150, receiving as much for the horse as for the 2 cows. What did he receive for each animal sold?

23. A man earns $13 a week and spends $5. In what time does he save $24?

24. A man pays $25 a month for his board and 12\frac{1}{2}$ for the board of his son. When the son's board has cost $50, what has been the cost of the man's?

25. A wagon wheel 6 ft. in circumference turns round 250 times in traveling between two houses. How far apart are they?

26. 1500 ft. is the distance between two houses. How many times does a wheel 6 ft. in circumference turn in passing over this distance?

27. 1500 is the number of feet between two houses, and 250 is the number of times a wheel

turns in passing over this distance. $\frac{1500}{250}$ equals the number of feet in what?

28. If 6 in. were added to the length of a line, the sum would equal twice its length. What is its length?

29. 17 in. is 1 in. more than twice the length of a pencil. What is its length?

30. If you wish to cut a cube into three equal layers, how many cuts must you make?

31. A square piece of ground is 135 ft. long. What is its perimeter?

32. What is the difference in the perimeter of a field 40 by 40 and the perimeter of a field 40 by 30?

33. What is the difference in the area of a rectangle containing 4 sq. ft. and a rectangle 4 ft. sq.? What is the difference in the perimeter of these rectangles?

34. A typewriter wrote 56 letters one day and $\frac{7}{8}$ as many the next. How many did she write in the two days?

35. There are 200 lb. in a barrel of pork. At $4\frac{1}{2}¢$ a pound, what is the cost of 4 bbl. of pork?

36. At \$2.40 a bushel, what part of a bushel of filberts can you buy for \$.80?

37. If the line b equals $\frac{3}{4}$ of the line a, what is the ratio of a to b?

If b is 216 ft., what is a?

The difference between these lines equals what part of a? what part of b?

38. Two rectangles are drawn so that you can make them equal by erasing $\frac{1}{3}$ of the larger. What is their ratio?

39. Two rectangles are drawn so that you can make them equal by adding to the smaller an amount equal to $\frac{1}{4}$ of the larger. What is the relation of the rectangles?

40. a The line a represents the amount paid for fruit. Draw a line to represent the selling price if the relation of the selling price to the cost is $\frac{5}{4}$; if it is $\frac{4}{3}$; if it is $\frac{3}{4}$; if it is $\frac{2}{3}$.

In each case the gain or loss equals what part of the cost?

41. From problem 40 make other problems, thus: If the cost was \$4, what was the selling price at a gain of $\frac{1}{4}$?

42. A man sold a cow worth \$35 at a loss of $\frac{1}{5}$. For what price did he sell her?

If he had sold at a profit of $\frac{1}{5}$, for what would he have sold?

43. The cost of a rocking chair equals how many thirds of $\frac{3}{8}$ of its cost?

44. A man bought a hat for \$3 and sold it for \$5. The gain equaled how many thirds of the cost of the hat?

45. A boy sold a knife for \$.60 and gained $\frac{1}{5}$. What was the ratio of the cost to the selling price? What was the cost?

46. A man sold a lot for $600 and lost $\frac{1}{4}$ of what he paid for it. What was the ratio of the buying price to the selling price? For what did he buy the lot?

Had he sold at a gain of $\frac{1}{4}$, how much would he have received?

47. $7 equals how many thirds of $3? What is the ratio of $\frac{7}{2}$ to $\frac{3}{2}$?

$\frac{7}{4}$ equals how many thirds of $\frac{3}{4}$? $\frac{3}{4}$ equals what part of $\frac{7}{4}$?

What is the relation of $3\frac{1}{2}$ in. to 2 in.? The weight of $3\frac{1}{2}$ lb. is how many times the weight of $\frac{1}{5}$ of $2\frac{1}{2}$ lb.? Compare $1\frac{1}{2}$ yd. with $3\frac{1}{2}$ yd.; $3\frac{1}{2}$ yd. with $1\frac{1}{2}$ yd.

48. If $1\frac{1}{2}$ yd. of lace cost $.30, what is the cost of $3\frac{1}{2}$ yd.?

49. If $3\frac{1}{2}$ yd. of cashmere cost $3.50, what is the cost of $1\frac{1}{2}$ yd.?

50. If $2\frac{1}{2}$ lb. of prunes cost $.25, what does 1 lb. cost?

51. If 6 yd. of flannel cost $4.80, what do 14 yd. cost?

52. If $\frac{2}{3}$ lb. of coffee costs $.20, what do $1\frac{2}{3}$ lb. cost?

53. If 3 lb. of sugar cost $.16, why may I infer that $1\frac{1}{2}$ lb. cost $\frac{1}{2}$ of $.16?

54. If $1\frac{1}{4}$ rows of potatoes yield 27 bu., how many bushels will $2\frac{1}{2}$ rows yield?

55. If a man earns $30 in 5 wk., in how many weeks does he earn $72?

56. If 7 yd. of velvet cost $21, what is the cost of 5 yd.?

57. If 20 acres of land cost $480, what do 27 acres cost?

58. Charles worked $\frac{3}{4}$ dy. for Mr. Brown and $1\frac{1}{2}$ dy. for Mr. Jones. He received a dollar a day. How much money did he earn?

59. A wagon bed 2 ft. deep holds 28 bu. of corn. If its sides be raised 1 ft., what will it then hold? if the sides be raised 6 in.? 9 in.?

60. The water in a cistern is 9 ft. deep. If enough runs in to raise it 3 ft., it will contain 96 bbl. How much did it contain when 9 ft. deep?

61. At $1 a bushel, what part of a bushel of berries can you buy for $.87$\frac{1}{2}$?

62. How many pounds of cheese at $12\frac{1}{2}¢$ a pound can you buy for $1?

63. At $12\frac{1}{2}¢$ a pint, how many pints of berries can you buy for $.75?

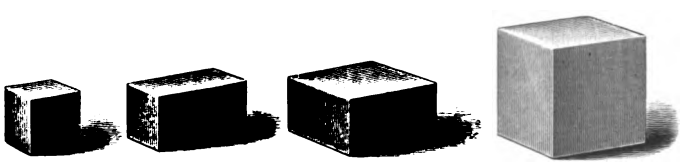
64. If 7 suits of clothes cost 87\frac{1}{2}$, what will 5 suits cost? 3 suits? 2 suits?

65. At 9¢ a yard, how many yards of cambric can you buy for $2.16?

PART III.

"Even the highest ideas are slowly and gradually developed from the accumulation of sensual experience." — Virchow.

"When the understanding is once stored with these simple ideas, it has the power to repeat, compare, and unite them, even to an almost infinite variety and so can make at pleasure new complex ideas. But it is not in the power of the most exalted wit or enlarged understanding to invent or frame one new single idea in the mind not taken in by the ways afore-mentioned." — Locke.



Place the cubes 2″ long, 4″ long, and the solids 2″ by 2″ by 4″ and 2″ by 4″ by 4″ where they can be handled.

Ratios. — 1. Tell all you can about the ratios of these units.

Ex.: 2 is the ratio of *c* to *b*.

2. *d* can be cut into how many *c*'s? into how many *b*'s? how many *a*'s?

3. Observe the units and find the ratio 1 as often as you can.[1]

4. Name the units whose ratio is 2.

Ex.: 2 is the ratio of c to b, of $\frac{1}{2}$ of c to $\frac{1}{2}$ of b, etc.

5. Name the units whose ratio is 3; whose ratio is 4.

6. Name the units whose ratio is $\frac{1}{2}$; whose ratio is $\frac{1}{4}$; whose ratio is $\frac{1}{8}$.

Ex.: $\frac{1}{8}$ is the ratio of a to $\frac{3}{4}$ of c.

7. Call c 1. What is the name of each of the other units?

8. Call b 1. What is the name of each?

9. Call d 1. What is the name of each?

10. The sum of b and c equals what part of d?

11. If c weighs 10 lb., what is the weight of each of the others?

12. If d represent 64 cu. ft. of wood, each of the other units represents what part of 64 cu. ft.?

13. If c holds 7 gal., what does each of the others hold?

Ex.: a holds $\frac{1}{4}$ of 7 gal.

14. Review again and again.

1. If the length of a is 1, what is the length of each of the other units?

What is the width of each? the height?

2. If a is the cube of 1, d is the cube of what?

3. The cube of 1 equals what part of the cube of 2?

4. The cube of 2 equals how many times the cube of 1?

5. A cube 1 ft. long equals what part of a cube 2 ft. long?

6. A cube 12″ long equals what part of a cube 24″ long?

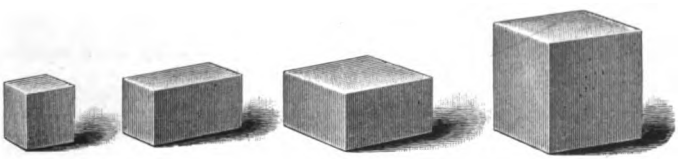
7. A cube 2″ long equals what part of a cube 4″ long?

[1] 1 is the ratio of $\frac{1}{2}$ of b to a, of $\frac{1}{2}$ of c to a, of $\frac{1}{8}$ of d to a, of $\frac{1}{2}$ of c to b, of $\frac{1}{4}$ of d to b, of $\frac{1}{4}$ of d to c, of b to 2 a's, of c to 4 a's, of c to 2 b's, of d to 8 a's, of d to 4 b's, of d to 2 c's.

8. A cube 4″ long equals how many times a cube 2″ long?

9. Find other cubes in which the ratio of the length of one to the length of the other is 2.

10. What is the ratio of a cube whose edge is x to one whose edge is $2x$? of a cube whose edge is $2x$ to one whose edge is $4x$?



Ratios of Cubes. — 1. Observe a cube.

Can a cube be cut into 2 equal cubes?

Can a cube be cut into 3 equal cubes? 5? 6?

2. Can a cube be cut into 4 equal cubes? 7? 8?

3. If the edge of a cube is 2, what are the dimensions of each of the 4 equal rectangular prisms into which it may be cut? Are the parts cubes? Why not?

4. What is the least number of equal cubes into which any cube can be cut?

5. Show how you would cut a cube to separate it into 8 equal cubes.

6. Find a cube equal to $\frac{1}{8}$ of another cube.

If the cube of 2 weighs 72 oz., what is the weight of the cube of 1?

7. If a gold cube whose length is 1 is worth $100, what is the value of a gold cube whose length is 2? whose length is $\frac{1}{2}$?

Give each of the pupils three squares.

Ratios of Squares. — 1. Can you cut a square into 2 equal squares? Try.

2. Can you cut a square into 3 equal squares? Try.

3. Can you cut a square into 4 equal squares? Try.

4. What is the least number of equal squares into which a square can be cut?

5. Draw a square foot on the blackboard. Mark it off into the least number of equal squares.

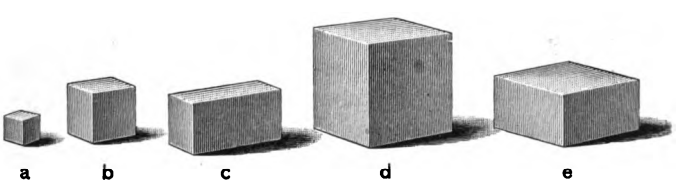
What is the length of each?

6. What is the ratio of a square $\frac{1}{2}$ ft. long to a square 1 ft. long?

7. What is the ratio of a square 1 ft. long to a square 6 in. long?

8. If there are 40 yd. of carpet in the square of 2, how many yards are there in the square of 1?

9. If you pay $\$x$ for a square rug whose edge is 1, what ought you to pay for a square rug whose edge is 2?



Place an inch cube, a 2-in. cube, a 4-in. cube, and a solid 2″ by 2″ by 4″ and another 2″ by 4″ by 4″ where they can be seen and handled.

Ratios. — 1. Tell all you can about the relations of these units.

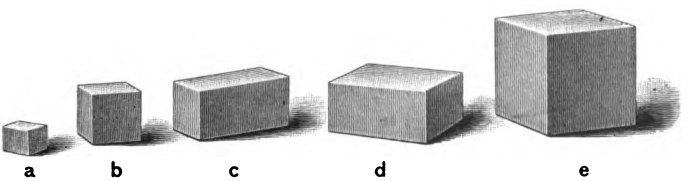
2. If a is 1 by 1 by 1, what are the dimensions of each of the other units?

3. Make sentences like this: A solid 2 by 2 by 4 equals $\frac{1}{2}$ of a solid 2 by 4 by 4.

4. What is the relation of b to each of the other units?
5. If b is 1, what is each of the other units?
6. If b is 8, a equals what part of 8?

How many 8's in c? in d? in e?

7. If b is 8, what is the name of each of the other units?
8. Compare 8 with 1; with 16; with 32; with 64.
9. Compare 16 with each; 32 with each; 64 with each.



1. If e is 1, what is each of the other units?
2. How many 64ths in the $\frac{1}{8}$? in the $\frac{1}{4}$? in the $\frac{1}{2}$? in the 1?
3. Compare $\frac{1}{8}$ with each of the other units.
4. Compare $\frac{1}{4}$ with each; $\frac{1}{2}$ with each; $\frac{3}{4}$ with each.
5. If a is a cubic inch, how many cubic inches in each of the other solids?
6. If b is a 2-in. cube, what part of a 2-in. cube is found in a?
7. Into how many 2-in. cubes can c be cut? into how many can d be cut?
8. How high a post can you make of the 2-in. cubes into which a 4-in. cube can be cut?
9. Compare 2 cu. in. with an inch cube.
10. How many cubes $\frac{1}{2}$ in. long can be cut from a cubic inch?
11. Compare a 2-in. cube with 2 cu. in.
12. Compare a $\frac{1}{2}$-in. cube with 2 cu. in.
13. How many $\frac{1}{2}$-in. cubes in a cubic inch? in a 2-in. cube?

14. How many 1-in. sq. can be cut from a paper 3 in. long 2 in. wide? how many $\frac{1}{2}$-in. sq.?

15. A 4-in sq. can be cut into how many 2-in. sq.?

16. If a gold cube, whose length is 1, is worth $\$x$, what is the value of a gold cube whose length is 2?

1. Observe the staffs. What ratios do you see?

2. What is the ratio of d to each of the others? of c? of b? of a?

3. What is the ratio of the shadow cast by d to the shadow cast by each of the others? of the shadow cast by c? by b? by a?

4. Assume 18 ft. to be the length of the shadow cast by d, and make sentences like this: $\frac{3}{2}$ is the ratio of the shadow of c to the shadow of d.

18 ft. is the shadow cast by d.

$\therefore \frac{3 \cdot 18 \text{ ft.}}{2}$ is the shadow cast by c.

5. Assume different lengths for the shadows of different staffs, and make sentences similar to those in problem 4.

6. If the shadow of e is $1\frac{1}{2}$ ft. long, what is the length of the shadow of each of the other staffs?

7. If 3 is the ratio of two shadows, what is the ratio of the height of the objects casting them?

8. If the ratio of two shadows is $\frac{2}{7}$, what is the ratio of the height of the objects casting them?

What is the relation of the taller object to the shorter one? of the longer shadow to the shorter one?

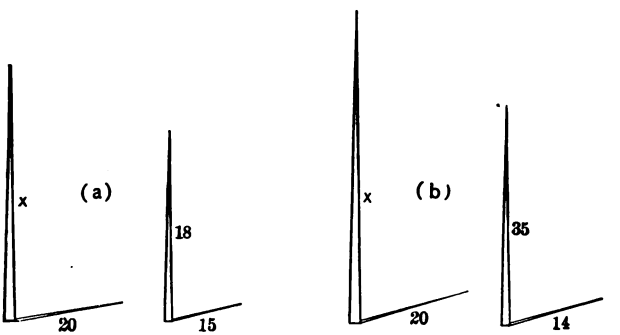
9. If a staff 10 ft. high casts a 15-ft. shadow, what is the height of a telegraph pole that casts a 60-ft. shadow?

What is the ratio of the *shadow* of the telegraph pole to the *shadow* of the staff?

4 is the ratio of what to the staff?

The ratio of the shadow of the telegraph pole to the shadow of the staff equals the ratio of what?

10. Draw staffs on the blackboard representing 2, 3, 4, 5, 6, and study the relations of the heights and of the shadows.



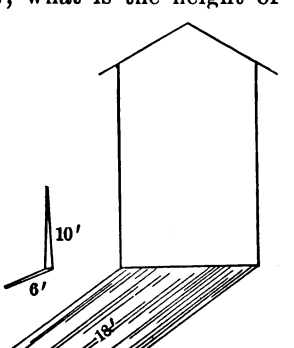
11. Find the height of x in a; of x in b.

12. 2 is the ratio of two shadows. What is the ratio of the objects casting them? If 35 ft. is the height of the object casting the shorter shadow, what is the height of the object casting the longer shadow?

13. Find the height of the house.

14. In what way can you indirectly find the height of the schoolhouse? of a telegraph pole?

15. Find the height of telegraph poles, flag staffs, and of other objects by this method.



Problems. — 1. If a pole 50 ft. high casts a shadow 70 ft. long, what is the length of a pole which casts a shadow of 30 ft.?

2. The ratio of the shadow cast by the pole A to the shadow cast by the pole B is 6. If B is 13 ft. long, how long is A?

3. The ratio of the longer of two staffs to the shorter is x. If y equals the number of feet in the shadow cast by the shorter staff, what equals the number of feet in the shadow cast by the longer staff?

4. The ratio of one staff to another is $\frac{1}{3}$. A equals the length of the shadow cast by the longer staff. What equals the length of the shadow cast by the shorter?

Equations. — 1. Use sticks and make 3 bundles of 7 each.

2. If you take one stick out of each of the bundles, how many do you take?

3. How many 3's are there in the bundles of 7?

4. Which is the more, $3 \cdot 7$'s or $7 \cdot 3$'s?

5. Show that $4 \cdot 10$'s equals $10 \cdot 4$'s.

6. O O O O O — How many rows of 5's do you see in the rectangle?
   O O O O O
   O O O O O — How many rows of 3's do you see in the same rectangle?

What is the ratio of $3 \cdot 5$'s to $5 \cdot 3$'s?

7. In a rectangle of $2 \cdot 9$'s, how many rows of 2's do you see?

8. Image a rectangle containing 5 rows of 9 sq. ft. How many rows of 5 sq. ft. do you see in this rectangle?

$5 \cdot 9$ sq. ft. equals what? $9 \cdot 5$ sq. ft. equals what?

What is the ratio of $5 \cdot 9$'s to $9 \cdot 5$'s?

9. Image, arranging in the form of a rectangle, 3 rows of \$7. In this rectangle, how many rows of \$3 do you see?

10. In a rectangle of 75 \$4, how many \$75 are there?

11. At \$5 a ton, how many \$5 will 18 tons of hay cost? how many \$18? Why?

12. At \$4 a barrel, what will 36 bbl. of flour cost? how many \$36?

13. At \$40 an acre, what will 272 acres of land cost?

14. At \$.02 a yard, what is the cost of 440 yd. of barbed-wire fencing?

Ratios. — 1. What is the ratio of 5 apples to $2\frac{1}{2}$ apples? of $5 \cdot 10$'s to $2\frac{1}{2} \cdot 10$'s?

2. What is the ratio of 1 bu. to 1 pk.? of 7 bu. to 7 pk.?

3. What is the ratio of 1 ft. to 1 yd.? of 4 ft. to 4 yd.?

4. What is the ratio of $7 \cdot 12$'s to $14 \cdot 12$'s? of $9 \cdot 8$'s to $9 \cdot 4$'s?

5. What is the ratio of $6 \cdot 7$'s to $3 \cdot 7$'s? of $5 \cdot 6$'s to $5 \cdot 3$'s?

6. What is the ratio of $3 \cdot 22$'s to $9 \cdot 22$'s? of $7 \cdot 35$'s to $7 \cdot 17\frac{1}{2}$?

7. What is the ratio of $12\frac{1}{2} \cdot 19$'s to $37\frac{1}{2} \cdot 19$'s? of $17 \cdot 56$'s to $17 \cdot 7$'s?

8. What is $\frac{1}{2}$ of $17 \cdot 6$'s?

$\frac{\overset{3}{17 \cdot \cancel{6}}}{\cancel{2}} = 51.$ What is the ratio of $17 \cdot 3$'s to $17 \cdot 6$'s? of 51 to $17 \cdot 6$'s?

9. What is $\frac{1}{4}$ of $18 \cdot 14$'s? What is $\frac{1}{4}$ of $16 \cdot 20$'s?

10. What is $\frac{1}{8}$ of $25 \cdot 24$'s? What is $\frac{1}{8}$ of $26 \cdot 25$'s?

1. What is $\frac{1}{2}$ of $\frac{1}{3}$ of $8 \cdot 45$'s?

$\frac{4 \cdot 15}{\cancel{2} \cdot \cancel{3}} = 60.$

$\frac{1}{3}$ of $8 \cdot 45$'s is 8 what?

What is $\frac{1}{2}$ of $8 \cdot 15$'s?

$4 \cdot 15$'s equals $\frac{1}{2}$ of $\frac{1}{3}$ of what?

What is the ratio of $8 \cdot 15$'s to $8 \cdot 45$'s?

What is the ratio of $4 \cdot 15$'s to $8 \cdot 15$'s?

2. What is $\frac{1}{8}$ of $\frac{1}{5}$ of 24 · 25's ?
3. What is $\frac{1}{13}$ of $\frac{1}{8}$ of 26 · 45's ?
4. What is $\frac{1}{11}$ of $\frac{1}{8}$ of 35 · 44's ?
5. What is $\frac{1}{2}$ of $\frac{1}{8}$ of 7 · 25's ?

$$\frac{7 \cdot \overset{4}{\cancel{25}}}{\underset{2}{\cancel{2}} \cdot \underset{1}{\cancel{3}}} = 29\tfrac{1}{6}.$$

What is $\frac{1}{3}$ of 7 · 25's ?
What is $\frac{1}{2}$ of 7 · $8\frac{1}{3}$?
7 · $4\frac{1}{6}$ equal what ?

6. What is $\frac{1}{4}$ of $\frac{1}{8}$ of 22 · 12's ?
7. What is $\frac{1}{8}$ of $\frac{1}{4}$ of 36 · 19's ?
8. What is $\frac{1}{3}$ of $\frac{1}{5}$ of 11 · 25's ?
9. What is $\frac{1}{7}$ of $\frac{1}{5}$ of 25 · 31's ?
10. What is $\frac{1}{8}$ of $\frac{1}{4}$ of 5 · 17's ?

1. What is $\frac{1}{12}$ of 440 ?[1]

$$\frac{\overset{110}{\cancel{440}}}{\underset{3}{\cancel{12}}} = 36\tfrac{2}{3}.$$

What is $\frac{1}{4}$ of 440 ?
What is the ratio of 110 to 440 ?
What part of $\frac{1}{4}$ equals $\frac{1}{12}$?
Then $\frac{1}{3}$ of 110 equals what part of 440 ?

2. What is $\frac{1}{15}$ of 845 ?

$$\frac{\overset{169}{\cancel{845}}}{\underset{3}{\cancel{15}}} = 56\tfrac{1}{3}.$$

What is $\frac{1}{5}$ of 845 ? What is the ratio of 169 to 845 ?
What part of $\frac{1}{5}$ equals $\frac{1}{15}$?
Then $\frac{1}{3}$ of 169 equals what part of 845 ?

3. What is $\frac{1}{16}$ of 248 ft. ?
4. What is $\frac{1}{37}$ of 269 hr. ?
5. What is $\frac{1}{9}$ of 15 · 35's ?
6. What is $\frac{1}{6}$ of 75 · 8's ?
7. What is $\frac{1}{37}$ of 45 · 13's ?

[1] Upon the power of picturing the relations depends the power of thinking in symbols.

"You may have a symbol of eternity — if, first, which is wholly needful, you have an *idea* of eternity." — John Ruskin.

8. What is $\frac{1}{12}$ of 27 · 33's ?

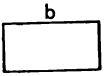
9. What is $\frac{1}{8}$ of $\frac{1}{15}$ of 24 · 65's ?

Ratios. — 1. What is the ratio of a to b ? of $\frac{1}{2}$ of a to $\frac{1}{2}$ of b ? of $\frac{1}{2}$ of b to $\frac{1}{2}$ of a ? of $\frac{1}{3}$ of b to $\frac{1}{3}$ of a ? of $\frac{2}{3}$ of b to $\frac{2}{3}$ of a ?

If we call a 1, what shall we call b ?

Show me $\frac{1}{3}$ of the 2 ; of the 1.

What is the ratio of $\frac{1}{3}$ of 2 to $\frac{1}{3}$ of 1 ?

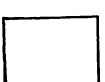


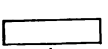
Then $\frac{1}{3}$ of 2 *equals* how many thirds of 1 ?

What is the ratio of $\frac{1}{3}$ of 2 tons of coal to $\frac{1}{3}$ of 1 ton ?

Then $\frac{1}{3}$ of 2 tons equals how many thirds of 1 ton ?

2. What is the ratio of a to b ? of b to a ? of $\frac{1}{2}$ of a to $\frac{1}{2}$ of b ? of $\frac{1}{3}$ of a to $\frac{1}{3}$ of b ?





What is the ratio of $\frac{1}{4}$ of a to $\frac{1}{4}$ of b ?

Then $\frac{1}{4}$ of a equals how many fourths of b ?

If we call a 3, what shall we call b ?

Show me $\frac{1}{2}$ of the 1 ; of the 3.

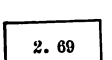
What is the ratio of $\frac{1}{2}$ of 3 to $\frac{1}{2}$ of 1 ?

Then $\frac{1}{2}$ of 3 equals how many halves of 1 ?

$\frac{1}{2}$ of 5 equals how many halves of 1 ? $\frac{1}{3}$ of 4 equals how many thirds of 1 ? What is the ratio of $\frac{1}{3}$ of 4 to $\frac{1}{3}$ of 1 ?

3. If a represents 2 · 69's, b represents what ? Measure off $\frac{1}{3}$ of the 2 · 69's and $\frac{1}{3}$ of 69.

What is the ratio of $\frac{1}{3}$ of 2 · 69's to $\frac{1}{3}$ of 69 ?



Then $\frac{1}{3}$ of 2 · 69's equals how many thirds of 69 ?

4. $\frac{1}{3}$ of 5 · 17's equals what ?

$\frac{2}{3}$ of 19 equals $\frac{1}{3}$ of what ?

$\frac{5}{7}$ of 92 equals $\frac{1}{7}$ of what ?

1. What is the ratio of a yard to a foot? of $\frac{1}{2}$ yd. to $\frac{1}{2}$ ft.? of $\frac{1}{3}$ yd. to $\frac{1}{3}$ ft.? of 5 yd. to 5 ft.? of $\frac{2}{3}$ yd. to $\frac{2}{3}$ ft.?

2. What is the ratio of a peck to a quart? of $\frac{1}{2}$ pk. to $\frac{1}{2}$ qt.? of $\frac{1}{3}$ pk. to $\frac{1}{3}$ qt.? of $\frac{1}{7}$ pk. to $\frac{1}{7}$ qt.?

3. Show by drawing the part of 1 ft. that equals $\frac{1}{4}$ of 3 ft.

4. If the ratio of $\frac{1}{4}$ of 3 to $\frac{1}{4}$ of 1 is 3, $\frac{1}{4}$ of 3 equals how many fourths of 1?

5. If 3 apples are divided equally among 4 boys, what part of 1 apple does each boy receive?

6. If 3 yd. of cloth are cut into 4 equal pieces, what part of a yard is each piece?

7. John has $\frac{1}{3}$ of \$2 and James $\frac{2}{3}$ of \$1. How many cents has each?

8. What is the ratio of 4 to 1? of $\frac{1}{5}$ of 4 to $\frac{1}{5}$ of 1?

Then $\frac{1}{5}$ of 4 equals how many fifths of 1?

9. I bought $\frac{1}{3}$ of a remnant of 4 yd. What part of a yard did I buy?

What is the ratio of $\frac{1}{3}$ of 1 yd. to $\frac{1}{3}$ of 4 yd.?

10. What is the ratio of 7 to 1? of $\frac{1}{4}$ of 7 to $\frac{1}{4}$ of 1?

Then $\frac{1}{4}$ of 7 equals how many fourths of 1?

What is the ratio of $\frac{1}{4}$ of 7 to $\frac{7}{4}$ of 1?

11. What is the ratio of 3 to 1? of any part of 3 to a corresponding part of 1?

Then $\frac{1}{5}$ of 3 equals how many fifths of 1?

$\frac{1}{7}$ of 3 equals how many sevenths of 1?

12. What is the ratio of $3 \cdot 12$'s to $1 \cdot 12$? of $\frac{1}{4}$ of $3 \cdot 12$'s to $\frac{1}{4}$ of $1 \cdot 12$? of $\frac{1}{7}$ of $3 \cdot 12$'s to $\frac{1}{7}$ of $1 \cdot 12$?

Then $\frac{1}{7}$ of 3 12's equals how many sevenths of 12?

13. What is the ratio of $5 \cdot 13$'s to 13? of $\frac{1}{7}$ of $5 \cdot 13$'s to $\frac{1}{7}$ of 13?

Then $\frac{1}{7}$ of $5 \cdot 13$'s equals how many sevenths of 13?

14. If $\frac{1}{3}$ of 4 equals $\frac{4}{5}$ of 1, $\frac{2}{3}$ of 4 equals how many $\frac{4}{5}$ of 1?

15. If $\frac{1}{4}$ of 12 equals $\frac{1}{7}$ of 1, then $\frac{5}{4}$ of 12 equals how many $\frac{1}{7}$ of 1?

16. If $\frac{1}{4}$ of 32 equals $\frac{3}{4}$ of 1, $\frac{5}{4}$ of 32 equals how many $\frac{3}{4}$?

$\frac{5 \cdot 32}{4} = ?$ $\frac{1}{4}$ of $5 \cdot 32$'s equals how many fourths of 32?

17. $\frac{1}{2}$ of $5 \cdot 28$'s equals how many halves of 28?

$\frac{5}{2}$ of 28 equals $\frac{1}{2}$ of how many 28's?

18. $\frac{5}{2}$ of \$72 equals $\frac{1}{2}$ of how many \$72?

$\frac{5 \cdot 72}{2} = 180.$ What is $\frac{1}{2}$ of $5 \cdot 72$'s?

What is the ratio of $5 \cdot 36$'s to $5 \cdot 72$'s?

of 180 to 5 72's? \$180 equals $\frac{5}{2}$ of what?

19. If 7 yd. of ribbon cost 84¢, 3 yd. will cost $\frac{1}{7}$ of how many 84¢?

$\frac{3 \cdot 84¢}{7} = ?$

20. If a man walks 29 mi. in 9 hr., how far does he walk in 7 hr.?

21. If 4 lb. of sugar cost 91¢, what will 25 lb. cost?

22. If $\frac{4}{5}$ yd. of ribbon costs 32¢, what will $\frac{3}{5}$ yd. cost?

23. What is .7 of 548 mi.?

$\frac{7 \cdot 548}{10} = ?$ or $7 \ 54.8 = ?$

24. What is .9 of \$275? of 5840 dy.?

25. What is .03 of 6400 ft.?

A pupil is not ready to solve a problem until he can repeat it and represent the conditions.

26. A farmer sold 45 sheep, which was $\frac{5}{8}$ of his flock. How many sheep had he before the sale?

What is the ratio of the flock to the part he sold?

27. 120 acres equal $\frac{3}{4}$ of the number of acres in a farm. How many acres in the farm?

28. A man paid \$24 interest, which equaled $\frac{6}{100}$ of the amount he borrowed. How much did he borrow?

29. A man sold $\frac{3}{8}$ of his farm and had 60 acres left. What was the ratio of the farm to 60 acres?

How many acres did he own at first?

30. Mr. Jones invested $\frac{3}{7}$ of his money and had \$64 left. What was the ratio of all his money to \$64? How much had he at first?

31. At \$.96 for $\frac{4}{7}$ yd. of lace, what will 1 yd. cost?

32. If 17 lamps cost \$51, what will 21 cost?

33. If 5 dictionaries cost \$40, what will 11 cost?

34. If $\frac{5}{7}$ of a basket of grapes costs 60¢, what will a basket cost?

35. 824 is the sum of 6 equal units. What is one of the units?

36. A boat runs 12 mi. an hour down stream. After running down stream 6 hr., it takes it 8 hr. to return. What is the rate per hour up stream?

37. A man bought 10 acres of land at \$20 an acre and 5 acres at \$25 an acre. What was the average price per acre?

38. A merchant bought 30 bbl. of flour for \$120 and sold it for \$210. What was the gain per barrel?

39. A man earns \$1200 a year and spends \$60 a month. What does he save each month?

Ratios. — What is the ratio of a to b? of b to a?

	a	b		a	b
1.	3 ft.	2 yd.	8.	7 bu. 3 pk.	2 bu. 2 pk.
2.	9 in.	1 ft.	9.	$7\frac{1}{2}$	6
3.	1 score	15 things	10.	$3\frac{1}{2}$	$3\frac{1}{4}$
4.	2 wk.	12 dy.	11.	2-in. cube	1-in. cube
5.	13 cu. ft.	1 cu. yd.	12.	2-in. cube	4-in. cube
6.	1 lb.	12 oz.	13.	6-in. cube	3-in. cube
7.	$2\frac{1}{2}$	7	14.	1 gal.	17 cu. in.

15.	6-in. cube	1728 cu. in.	18.	$5\frac{3}{8}$	2
16.	2-in. sq.	1-in. sq.	19.	$3\frac{1}{7}$	$4\frac{2}{7}$
17.	$\frac{1}{2}$-in. sq.	2-in. sq.	20.	$6\frac{1}{4}$	$3\frac{1}{2}$
21.	A rect. 2 ft. by 5 ft.		A rect. 8 ft. by 5 ft.		
22.	A rect. solid 7 by 12 by 15		A sq. $3\frac{1}{3}$ by 12 by 15		

1. What is the ratio of 6972 to 35?

$$\frac{996}{\cancel{6972}\ \cancel{35}\ 5} = 199\tfrac{1}{5}, \text{ or}$$

$$35\overline{)6972}\ (199\tfrac{1}{5}$$
$$\underline{35}$$
$$347$$
$$\underline{315}$$
$$322$$
$$\underline{315}$$
$$7$$

What is $\frac{1}{7}$ of 6972? of 35?
What is the ratio of 996 to 5?
What, then, is the ratio of 6972 to 35?

What is the ratio of a to b?

	a	b		a	b
2.	4684	32	8.	92,635	92
3.	9633	63	9.	9565	64
4.	6792	52	10.	36,246	189
5.	36,872	72	11.	53,004	398
6.	24,898	23	12.	62,346	254
7.	7842	23	13.	375,426	521

Problems. — 14. At \$176 each, how many carriages can you buy for \$12,672?

If 72 carriages cost \$12,672, what is the cost of 1?

72 is the ratio of what to \$176?

15. At the rate of 108 mi. a day, in how many days will a ship sail 2646 mi.?

16. At \$75 an acre, how many acres of land can be bought for \$22,546?

17. \$3850 was paid for a herd of cattle at \$45 a head. What was the ratio of \$3850 to the price per head?

18. $1800 is the money a man spends in a given time, at the rate of $150 per month.

The ratio of $1800 to $150 equals the ratio of what to 1 mo.?

How long does $1800 last?

19. x equals the cost of a farm, y equals the cost of an acre. The ratio of x to y equals the ratio of what to 1 acre?

Ratios. — 1. What is the ratio of $24 \cdot \$17$ to $\$16$?

What is $\frac{1}{8}$ of $24 \cdot \$17$? of $\$16$?

What is the ratio of $\frac{1}{8}$ of $24 \cdot \$17$ to $\frac{1}{8}$ of $\$16$?

What, then, is the ratio of $24 \cdot \$17$ to $\$16$?

$\$16$ equals what part of $24 \cdot \$17$?

Into how many $\$16$ can $24 \cdot \$17$ be separated?

$$\frac{\overset{3}{\cancel{24}} \cdot \$17}{\$\cancel{16}_{2}} = \frac{51}{2} = 25\frac{1}{2}.$$

What is the ratio of a to b? of b to a?

	a	b		a	b
2.	$19 \cdot 84$'s	72	5.	$256 \cdot 12$'s	36
3.	$84 \cdot 19$'s	72	6.	$42 \cdot 18$'s	$6 \cdot 42$'s
4.	$63 \cdot 51$'s	34	7.	$420 \cdot 45$'s	$40 \cdot 75$'s

Problems. — 8. For 640 acres of land, valued at $20 an acre, a man trades land valued at $75 an acre. How many acres at $75 were required?

9. 25 lots at $50 each were exchanged for coal at $5 a ton. How many tons were required?

10. 3 bolts of muslin, each containing 27 yards at $.06 a yard, was paid for with butter at $.18 per pound. How many pounds did it take?

11. How many tons of hay at $9 a ton must be given for 30 cows at $28 each?

Ratios. — 1. 3 is the ratio of what to 4 ?

2. 2 is the ratio of what to $580 ?

3. 7 is the ratio of what to 1273 ft. ?

4. 5 is the ratio of what to 472 oz.?

5. $\frac{1}{2}$ is the ratio of what to 9876 ?

6. $\frac{3}{4}$ is the ratio of what to 7854 ?

7. $\frac{3}{5}$ is the ratio of what to 249 ?

8. $1\frac{1}{2}$ is the ratio of what to 782 ?

9. What is the ratio of 782 to a unit equal to $\frac{3}{5}$ of 782?

10. 29 is the ratio of what to 546 ?

11. 45 is the ratio of what to 6452 ?

12. 186 is the ratio of what to 793 ?

13. 457 is the ratio of what to 894 ?

Problems. — 14. At $650 each, what is the cost of 32 city lots ?

32 is the ratio of what to 650 ?

What is the ratio of 20,800 to 650 ?

15. If one schoolroom seats 48 pupils, how many will a building of 16 such rooms seat ?

What is the ratio of the number of pupils the building will seat to the number one room will seat ?

16. At 45 bu. to the acre, how many bushels of wheat are raised on 2540 acres ?

17. At $52 an acre, what is the cost of 176 acres of land ?

18. At the rate of 38 mi. a day, how many miles are traveled in 154 dy. ?

154 is the ratio of what to 38 ?

Number Ratios. — 1. Draw a line 1 yd. long.

What is the *number* of yards in the line ? what is the *number* of feet ?

What is the ratio of the *number* of feet to the *number* of yards in the line ?

What is the ratio of the *number* of yards to the *number* of feet?

2. Draw a line 2 yd. long.

What is the *number* of feet in the line? of yards?

What is the ratio of the *number* of feet to the *number* of yards? of the *number* of yards to the *number* of feet?

3. What is the ratio of the *number* of feet in any line to the *number* of yards?

If x is the *number* of yards in a line, what is the *number* of feet?

4. What is the ratio of the *number* of dimes in a dollar to the *number* of cents?

What is the ratio of the *number* of \$5 in x to the *number* of dollars in x?

5. Take 2 pieces of paper, each 2 in. by 4 in. Cut one into 2-in. sq., the other into square inches.

What is the ratio of the *number* of 2-in. sq. to the *number* of square inches?

6. What is the ratio of the *number* of 2-in. sq. in any surface to the *number* of square inches?

If y is the *number* of square inches, what is the *number* of 2-in. sq.?

7. What is the ratio of the *number* of square yards in the walls of a room to the *number* of square feet?

If 189 is the *number* of square feet, what equals the *number* of square yards? $\frac{1}{9}$ of 189 equals the *number* of what?

8. If $13 \cdot 18$'s equals the *number* of square feet in the surface of a floor, what equals the *number* of square yards in it?

$$\frac{13 \cdot 18}{9} = \text{number.} \quad (\text{Read, } \tfrac{1}{9} \text{ of } 13 \cdot 18\text{'s.})$$

9. The *number* of 2-in. cubes in a piece of wood equals what part of the *number* of inch cubes in it?

10. The *number* of 2-in. cubes in a block 6 in. by 9 in. by 15 in. equals what part of the *number* of inch cubes in it?

$$\frac{6 \cdot 9 \cdot 15}{8} = \text{number}.$$

11. What is the ratio of the *number* of gallons in a can to the *number* of quarts in it?

12. If 84 is the *number* of quarts in a can, what is the *number* of gallons?

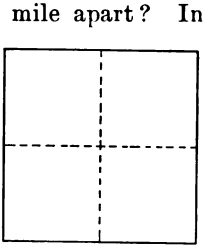
13. If x is the *number* of quarts in a can, what equals the *number* of gallons?

14. If y is the *number* of gallons in a tank, what equals the *number* of quarts?

15. Make sentences like this: If 72 is the *number* of yards in a line, $3 \cdot 72$'s equals the *number* of feet in it.

16. Make sentences like this: $\frac{1}{9}$ of 2433 equals the *number* of square yards in the walls of a room, if 2433 is the *number* of square feet.

Square Mile — Section. — 1. Do you know places that are a mile apart? What places are a mile apart? In what time do you think you can walk a mile? The distance from your home to the schoolhouse equals what part of a mile?



A Square Mile.

2. A section of land is a square mile.

What is the length of one side of a section? what is the width?

3. If you can walk a mile in 15 min., in what time can you walk around a section?

4. There are 640 acres in a section. How many acres in a quarter section?

5. How many 160 acres are there in a section?

6. How many 160 acres are there in a half section?

7. 160 acres equal what part of a section?

8. $2 \cdot 160$ acres equals what part of a section?

9. What is the length of a quarter section?

10. How many miles is it around a quarter section?

11. A piece of land $\frac{1}{8}$ mi. long and $\frac{1}{2}$ mi. wide equals what part of a section?

12. A piece of land 1 mi. long and $\frac{1}{2}$ mi. wide equals what part of a section?

13. Place a finger on the northeast quarter section.

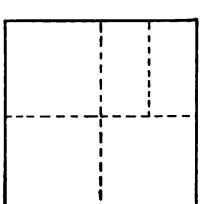
14. Place a finger on the northeast corner of the northeast quarter.

15. Place a finger on the southwest corner of the northeast quarter.

Draw lines in the section from north to south, dividing each 160 acres into two equal parts.

1. How many acres are there in a quarter section?

2. How many 80 acres are there in a quarter section?



3. How many 80 acres are there in a half section?

4. How many 80 acres are there in a section?

5. $2 \cdot 80$ acres equals what part of a section?

6. $4 \cdot 80$ acres equals what part of a section?

7. A piece of land $\frac{1}{2}$ mi. long and $\frac{1}{4}$ mi. wide contains how many acres?

8. How many 80-acre farms are there in a piece of land 1 mi. long and $\frac{1}{2}$ mi. wide?

9. How many 80-acre farms are there in a piece of land 1 mi. long and $\frac{3}{4}$ mi. wide?

10. A piece of land $\frac{3}{4}$ mi. long and $\frac{1}{2}$ mi. wide contains how many 80-acre farms?

11. How many acres are there in the west $\frac{1}{2}$ of the northwest $\frac{1}{4}$ of the section?

Draw lines in the section from west to east, dividing each 80 into two equal parts.

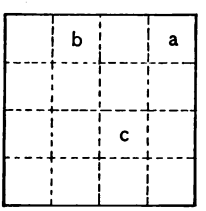
1. Write ten sentences about the section separated into 40-acre farms.

2. How many 40 acres are there in a section?

3. How many 40 acres are there in a half section?

4. How many 40 acres are there in a quarter section?

5. $4 \cdot 40$ acres equals what part of a section?



6. $8 \cdot 40$ acres equals what part of a section?

7. How many 40's are there in $4 \cdot 80$'s?

8. A piece of land $\frac{1}{2}$ mi. long and $\frac{1}{2}$ mi. wide contains how many 40's?

9. How many 40's touch the 40 in the southwest corner of the section?

10. How many 40's touch the northeast $\frac{1}{4}$ of the southwest $\frac{1}{4}$ of the section?

11. How many acres are there in the smallest square farm in the section?

12. How many squares of different sizes, or dimensions, do you see in the section?

13. Express in parts of a mile the dimensions of the different squares you see in the section.

14. How many 40 acres are there in each of the different squares? 40 acres equal what part of each of the different squares in the section?

15. What is the length of a square farm of 40 acres?

16. What is the perimeter of a square of 40 acres?

17. How many rows of $4 \cdot 40$'s are there in the section?

18. How many 40's are there in a section?

19. How many $\frac{1}{4}$-mi. sq. are there in a section?

20. What is the ratio of a $\frac{1}{4}$-mi. sq. to a square mile?

21. The west $\frac{1}{2}$ of the northwest $\frac{1}{4}$ contains how many acres?

22. The northeast $\frac{1}{4}$ of the southwest $\frac{1}{4}$ contains how many acres?

23. Describe the 40 marked *a*.

24. Describe the 40 marked *c*.

25. Describe the 40 marked *b*.

26. Have pupils locate a schoolhouse, a farmhouse, etc., in a section and tell where they are. Have pupils describe position of farm in section, and have others find it.

A square farm of 40 acres is 80 rods long.

1. The distance around 40 acres is how many 80 rods?

2. How many 80 rods are there in the length of a quarter section?

3. How many 80 rods are there in the length of a section?

4. 1 mi. equals how many 80 rods?

5. How many 80 rods are there in the distance around a quarter section?

6. How many 80 rods would you travel in going by the road from the southwest corner of the section to the northeast corner?

7. How many 80 rods are there in the distance around a section?

8. How many 80 rods of fence will it require to enclose a square farm of 40 acres? a square farm of 160 acres?

9. A mile equals how many rods?

10. How many sections in a township 6 mi. square?

11. A square containing 40 acres is how many rods long?

12. A piece of land 80 rods long and 2 rods wide equals

what part of a square containing 40 acres? It equals how many acres?

How many square rods in an acre?

13. An alley is 2 rods wide. How long will it have to be to contain 1 acre?

14. How many acres in a piece of land 80 rods long and 42 rods wide?

15. How many acres in a piece of land 40 rods long and 28 rods wide?

16. How wide is a strip of land a mile long that contains an acre?

17. The number of acres in a farm equals what part of the number of square rods in it?

18. A farm 120 rods long and 90 rods wide contains how many acres?

$$\frac{120 \quad 90}{160} = \text{number.}$$

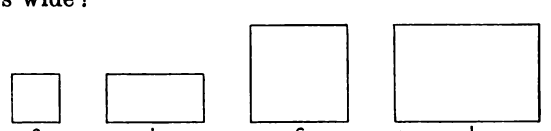
19. A mile equals how many rods?

There are 80 chains in a mile. How many rods in a chain?

20. There are 5280 ft. in a mile. How many feet in a chain?

21. 4 rods equal one chain. How many feet in a rod? How many yards in a rod?

22. How many acres in a farm 60 chains long and 40 chains wide?



Ratios. — 1. What ratios do you see?

2. What is the ratio of *b* to each? of *c*? of *d*?

3. Draw the units on the blackboard, making *a* a square foot.

4. If a represents a floor 10 ft. long, what is the length and the width of each of the other floors? What is the number of square feet in each?[1]

5. If x girls can sweep a in 3 min., in what time can they sweep each of the other floors?

6. If x girls can sweep c in 10 min., in what time can they sweep each of the others?

7. If x girls can sweep a in 3 min., what part of each of the other units can they sweep in the same time?

8. If 4 girls can sweep b in x min., how long will it take them to sweep each of the other floors?

9. If 4 girls sweep a in x min., 2 girls can sweep what part of each in x min.?

10. If 2 girls sweep $\frac{1}{2}$ of a in x min., in what time can they sweep a? in what time can they sweep each of the others?

11. If y girls can sweep $\frac{1}{2}$ of c in x min., in what time can they sweep c? in what time can they sweep each of the others?

12. What is the ratio of a to $\frac{1}{2}$ of a? of a to $\frac{1}{4}$ of a? to $\frac{3}{4}$ of a?

13. What is the ratio of each of the other units to $\frac{1}{2}$ of a? of $\frac{1}{2}$ of a to each of the others?

14. What is the ratio of $\frac{1}{4}$ of a to each of the other units? of each to $\frac{1}{4}$ of a?

15. What is the ratio of $\frac{3}{4}$ of a to each unit? of each unit to $\frac{3}{4}$ of a?

16. Represent other floors on the blackboard having the ratios of 1, 2, 4, 3, 9, and continue exercises similar to those above.

1. What is the ratio of the work that 8 boys can do to the work that 4 boys can do in the same time?

[1] See Teachers' Book, pp. 22, 23, 24.

What is the ratio of the work that 8 boys can do to the work that 6 boys can do in the same time?

2. What is the ratio of the work that 5 girls can do in x min. to the work that 2 girls can do in the same time?

What is the ratio of the floor of this room to $\frac{2}{5}$ of it?

What is the ratio of the time that it will take 2 girls to sweep this floor to the time it will take them to sweep $\frac{2}{5}$ of it?

3. What is the ratio of the time that it will take 3 men to build a wall to the time that it will take them to build $\frac{3}{4}$ of it?

4. What is the ratio of the number of yards of carpet necessary to carpet a floor to the number necessary to carpet $\frac{2}{3}$ of it?

5. What is the ratio of the time that it will take 3 pipes to fill a cistern to the time it will take them to fill $\frac{2}{5}$ of it?

6. What is the ratio of 75 to $\frac{1}{2}$ of 75? of 17 to $\frac{1}{2}$ of 17? of 25 to $\frac{3}{7}$ of 25? of $87\frac{1}{2}$ to $\frac{3}{4}$ of $87\frac{1}{2}$? of $\frac{4}{5}$ to $\frac{3}{4}$ of $\frac{4}{5}$?

Problems. — 1. If in x dy. a certain number of men can do 3 times the work required, in what time can they do the required work?

2. If the work that y men can do in x dy. equals $\frac{3}{5}$ of the work required, in what time can y men do the work required?

3. If the oil that passes through y pipes in x min. equals $\frac{2}{5}$ of the oil that a tank holds, in what time will y pipes fill the tank?

4. Ask and answer many questions like this: If x yd. of carpet equal $\frac{2}{3}$ of the carpet required, how many yards are required?

5. If 4 girls can do a piece of work in x min., in what time can 2 girls do it?

Represent the work that the 4 girls do in x min.

Show the part of the work that 2 girls do in x min.

What is the ratio of the work to be done to the work that 2 girls can do in x min.?

What ratios are equal?

6. If 4 girls can do a piece of work in x min., in what time can 3 girls do it?

Represent the work that 4 girls do in x min.

Show the part that 3 girls do in x min.

Show the work that the 3 girls have to do.

What is the ratio of the work to be done to the work that 3 girls can do in x min.?

What, then, is the ratio of the time to x min.?

7. Make problems and write questions about them similar to the following:

In what time can 3 girls do the work that 5 girls do in x min.?

What is the part of the work that 3 girls can do in x min.?

What is the ratio of the work to be done to the work that 3 girls can do in x min.?

What, then, is the time that it will take 3 girls to do the work that 5 girls do in x min.?

8. If 20 men in x dy. can do an amount of work equal to $\frac{2}{3}$ of the work required, in what time can they do the required work?

9. Make problems and write questions about them, thus:

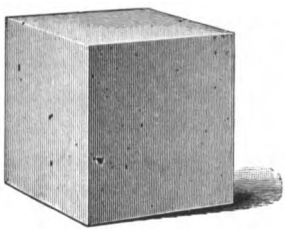
If 4 men earn \$$x$ in 20 dy., in what time can 6 men earn an equal amount?

What can 6 men earn in 20 dy.?

What is the ratio of \$$x$ to the money that 6 men can earn in 20 dy.?

What, then, is the time that it will take 6 men to earn \$$x$?

Make a cube 1 ft. long of tough check, or have it cut out of wood. Place the cube where it can be studied.



Cubic Foot. — 1. Write all you can about the cube 1 ft. long.

2. If the cube 1 ft. long is cut horizontally through the middle, what are the dimensions of each part?

What is the ratio of one of the parts to the other? of the cube to one of the parts? of one of the parts to the cube? of the sum of the two equal parts to the cube?

3. If you now cut the cube vertically through the middle, from right to left, what are the dimensions of one of the parts?

What ratios do you see?

What is the ratio of the cube to one of the parts? of the cube to three of the parts?

4. If you now cut the cube vertically through the middle, from front to back, what are the dimensions of the parts?

What is the shape of the parts?

What is the ratio of the cubic foot to one of these cubes? to 3 of them? to 5? to 7? to 8?

5. Write the directions for cutting a cube into 8 equal cubes.

Into how many cubes $\frac{1}{2}$ ft. long can a cubic foot be cut?

Into how many cubes $\frac{1}{2}$ in. long can a cubic inch be cut?

Into how many cubes $\frac{1}{2}$ yd. long can a cube 1 yd. long be cut?

Into how many cubes $\frac{x}{2}$ long can a cube x long be cut?

6. Cut a cubic foot horizontally into 3 equal layers. What are the dimensions of each layer?

Into how many cubes $\frac{1}{3}$ ft. long can one of these layers be cut?

Into how many cubes $\frac{1}{8}$ ft. long can a cubic foot be cut?

Into how many 4-in. cubes can a cubic foot be cut?

What is the ratio of a cubic foot to a cube $\frac{1}{3}$ ft. long? to 9 cubes each $\frac{1}{3}$ ft. long? to 15 cubes each $\frac{1}{3}$ ft. long?

7. Cut a cubic foot horizontally into 4 equal layers. What are the dimensions expressed in feet of one of these layers? What are the dimensions expressed in inches?

Into how many cubes 3 in. long can one of these layers be cut?

Into how many cubes $\frac{1}{4}$ ft. long can a cubic foot be cut?

How many 16's in 64? How many 3-in. cubes in a 6-in. cube? How many 8's in 64?

8. A layer 1 ft. by 1 ft. by $\frac{1}{8}$ ft. equals what part of a cubic foot?

A cubic foot can be cut into how many layers $\frac{1}{8}$ ft. thick?

One of these layers can be cut into how many prisms 12 in. by 2 in. by 2 in.?

Each of these prisms can be cut into how many cubes 2 in. long?

One of the layers can be cut into how many 2-in. cubes?

9. A cubic foot can be cut into how many layers 1 in. thick?

How many cuts must be made to separate the cube into 12 layers 1 in. thick?

Into how many prisms 1 ft. by 1 in. by 1 in. can a layer be cut?

Into how many cubic inches can one of the smaller prisms be cut?

Into how many rows of 12 cu. in. can a layer be cut?

Into how many cubic inches can a layer be cut?

Into how many cubic inches can a cubic foot be cut?

What is the ratio of 1728 cu. in. to a cubic foot?

1728 cu. in. equal how many 6-in. cubes? how many 4-in. cubes? how many 3-in. cubes? how many 2-in. cubes?

10. How many surfaces, edges, and points has a cube?

How many dimensions has a cube?

What is true of the three dimensions of a cube?

1. If the edge of a 6-in. cube is 1, what is the edge of a 12-in. cube?

What is the ratio of the cube of 2 to the cube of 1?

If 8 is the ratio of two cubes, what is the ratio of their edges?

If $\frac{1}{8}$ is the ratio of two cubes, what is the ratio of their edges?

2. If the edge of a 4-in. cube is 1, what is the edge of a 12-in. cube?

The ratio of the cube of 3 to the cube of 1 is what?

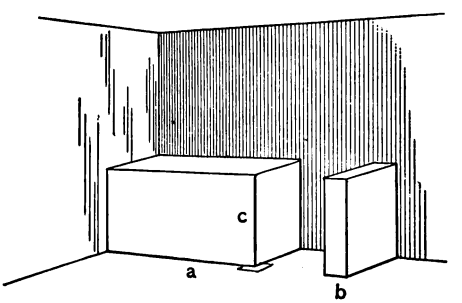
If 27 is the ratio of two cubes, what is the ratio of their edges?

If $\frac{1}{27}$ is the ratio of two cubes, what is the ratio of their edges?

3. The ratio of a cube 1 yd. long to a cube 1 ft. long is what?

A cubic yard equals how many cubic feet?

Show the size and dimensions of a cord of wood by making an outline in one corner of the room similar to the one in the diagram.



A Cord of Wood. — 1. Estimate dimensions. Measure. Write all you can about a cord.

2. If a represents a cord and b a cord foot, what is the ratio of a cord to a cord foot?

3. How many cubic feet in a cord foot? How many 16 cu. ft. in a cord?

4. What is the ratio of a cord to 16 cu. ft.? of 16 cu. ft. to a cord?

5. What is the ratio of a cord to a cubic foot?

6. A pile of wood 4 ft. long, 4 ft. wide, and 5 ft. high equals what part of a cord?

7. Give the dimensions of different piles of wood you can find in a cord.

8. A pile of wood 2 ft. long, 4 ft. high, and 4 ft. wide equals what part of a cord?

9. How many cord feet are there in a pile of wood 3 ft. long, 4 ft. high, and 4 ft. wide?

10. A pile of wood 8 ft. long, 4 ft. high, and 2 ft. wide equals what part of a cord?

11. A pile of wood 8 ft. long, 4 ft. high, and 1 ft. wide equals what part of a cord?

12. A pile of wood 8 ft. long, 4 ft. high, and 3 ft. wide equals what part of a cord?

13. How many cords of wood are there in a range 16 ft. long, 4 ft. high, and 4 ft. wide? How many 8 ft. long would the range have to be to have 3 cords in it?

14. How many cords of wood can be piled in a room 8 ft. in each dimension?

15. How many cords of 4-ft. wood can be piled with its ends against the walls of a room 12 ft. in each dimension? What are the dimensions of the unoccupied portion of the room?

16. If $\frac{1}{2}$ cord of wood is cut into 2-ft. sticks, how long a pile 4 ft. high will it make? How long a pile 2 ft. high will it make?

1. The number of cord feet in any pile of wood equals what part of the number of cubic feet? The number of cords equals what part of the number of cord feet?

2. $\frac{1}{8}$ of $\frac{1}{16}$ of the number of cubic feet in any pile of wood equals the number of what in the pile? Why?

3. If $4 \cdot 6 \cdot 74$ equals the number of cubic feet in a range of wood, what equals the number of cord feet? the number of cords?

$$\frac{4 \cdot 6 \cdot 74}{8 \cdot 16} = \text{number.}$$

4. How many cords in a range of wood 8 ft. high, 12 ft. wide, and 80 ft. long?

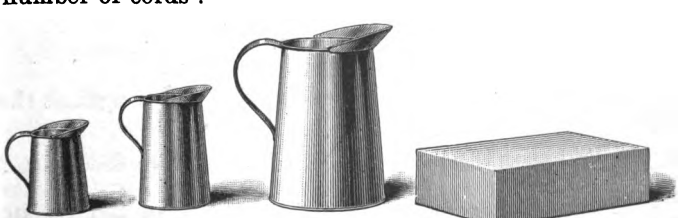
5. A pile of 4-ft. wood 12 ft. by 4 ft. equals how many half cords?

6. Make sentences like this: A pile of wood 20 ft. long, 4 ft. wide, and 4 ft. high contains $2\frac{1}{2}$ cords.

7. How many cords of wood in a pile 17 ft. long, 8 ft. wide, and 4 ft. high?

8. Write answers to problems similar to problem 7.

9. If a is the number of feet in the length of a pile of wood, b the number of feet in the width, and c the number in the height, what equals the number of cord feet? the number of cords?



Cut a prism of wood 3 in. by 7 in. by 11 in.

A Gallon. — 1. Study the prism and write what you can about it.

2. What is the length of the largest cube that can be cut out of the prism?

If there is no waste in cutting, how many 3-in. cubes can be cut?

How many cubic inches can be cut out of the piece or pieces left?

3. If the prism is cut into four equal parts, what are the dimensions of one of the parts?

How many cubic inches in the prism?

How many in one of the four equal parts?

What is the ratio of 231 to $57\frac{3}{4}$?

What is $\frac{1}{4}$ of $57\frac{3}{4}$?

What is the ratio of 231 to $28\frac{7}{8}$?

4. If you cut a prism 3 in. by 7 in. by 11 in. into four equal parts, one of the parts equals how many cubic inches?

If you cut one of the four equal parts into two equal parts, one of the two equal parts equals how many cubic inches?

Have a tinner make a tin pan 3 in. by 7 in. by 11 in. inside measure. Fill a gallon measure with water. Pour the water into the tin pan.

5. One gallon equals how many cubic inches?

Four quarts equal how many cubic inches?

One quart equals how many cubic inches?

Eight pints equal how many cubic inches?

One pint equals how many cubic inches?

6. How many cubic inches can you place against the sides of the tin pan?

7. What are the dimensions of the unoccupied space?

1. A gallon of water occupies a space of 231 cu. in. How many cubic inches in 6 gal.?

2. What is the ratio of the number of cubic inches in any tank to the number of gallons? of the number of gallons to the number of cubic inches?

3. 3465 cu. in. equal the space in a can. How many gallons of milk will it hold?

4. If the capacity of the can is 3465 cu. in., how many cubic feet does it contain?

5. The dimensions of a box in feet are 1 by 1 by 1. What are its dimensions in inches? A cubic foot equals how many cubic inches?

A gallon equals what part of 1728 cu. in.?

A Bushel. — 1. A bushel[1] equals $1\frac{1}{4}$ cu. ft. nearly. A cubic foot equals what part of a bushel?

17 cu. ft. equal what part of 17 bu.?

2. If 17 equals the number of cubic feet in a bin, what part of 17 equals the number of bushels in it?

3. There are 72 cu. ft. in a bin. What equals the number of bushels it will hold?

4. There are 90 bu. in a bin. What equals the number of cubic feet?

5. What is the ratio of the number of cubic feet in any bin to the number of bushels? of the number of bushels to the number of cubic feet?

6. x equals the number of cubic feet in a bin. What equals the number of bushels?

7. 80 equals the number of bushels a bin contains. $\frac{5 \cdot 80}{4}$ equals the number of what?

8. A bin contains 250 cu. ft. What equals the number of bushels it contains?

9. If $8 \cdot 6 \cdot 5$'s equals the number of cubic feet in a bin, what equals the number of bushels in it?

10. $7 \cdot 19 \cdot 20$'s equals the number of cubic feet. What equals the number of bushels?

11. If 160 equals the number of bushels a bin holds,

[1] A bushel equals 2150.4 cu. in.

what equals the number of cubic feet? if 400 equals the number of bushels? 300?

12. $6 \cdot 9 \cdot 10$'s equals the number of cubic feet in a bin. What equals the number of bushels?

$$\frac{4 \quad 6 \cdot 9 \cdot 10}{5} = \text{number.}$$

13. Pupils make and solve similar problems.

14. A cubical bin 5 ft. deep will hold how many bushels?

15. The contents of a box 3 ft. long and 2 ft. wide are 18 cu. ft. How high is the box?

16. There are 4 rows of $5 \cdot 2$-in. cubes in a box. What are the dimensions of the box?

17. A bin containing 30 cu. ft. is 3 ft. long and 2 ft. wide. How high is it?

18. A bin is 7 ft. long and 4 ft. wide. How deep must it be to contain 200 bu. of wheat?

19. What is the depth of a square tank 3 ft. long which contains 120 gal.?

1. A bushel equals 4 pk. A peck equals how many cubic inches?

2. A peck equals 8 qt. A quart equals how many cubic inches?

3. A gallon equals 231 cu. in. A liquid quart equals how many cubic inches?

4. What is the ratio of a liquid quart to a dry quart? of a dry quart to a liquid quart? of a dry pint to a liquid pint?

5. In liquid measure, a pint equals how many cubic inches?

6. In dry measure, a pint equals how many cubic inches?

7. What is the ratio of a liquid pint to a dry pint?

Problems. — 1. Measure the dimensions of the schoolroom. Give the result in feet in round numbers.

2. How many cubic feet can you place against the lateral surface of the schoolroom?

3. What are the dimensions of the unoccupied space?

4. Measure a room at home and give the dimensions to-morrow, and tell how many cubic feet you can place against the walls and the dimensions of the unoccupied space.

5. In a room 9 ft. long, 7 ft. wide, and 8 ft. high, how many walls 8 ft. by 9 ft.? how many 7 ft. by 8 ft.?

6. How many cubic feet can you place against the two walls 8 ft. by 9 ft.?

7. What are the dimensions of the unoccupied space?

8. How many cubic feet can you place against the walls of a room 8 ft. in each dimension?

9. What are the dimensions of the unoccupied space?

10. How many cubes 2 ft. long can you place against the walls of a room 12 ft. in each dimension?

11. What are the dimensions of the unoccupied space?

12. How many 2-in. tin boxes can you put into a box 6 in. long, 5 in. wide, and 4 in. high?

What are the dimensions of the unoccupied space?

13. If a box is 6 in. wide and 4 in. high, how long will it have to be to hold 25 2-in. cubes?

1. If the cost of 2 peaches equals the cost of 4 apples, how many peaches can you buy for the price of 48 apples?

2. What is the price per dozen when you can buy 3 oranges for 10¢?

3. At the rate of 3 for 7¢, how many peaches can I buy for 84¢?

4. What is the cost of a dozen bananas at the rate of 8 for 6¢?

5. A case of strawberries containing 24 boxes sells at 3 boxes for 25¢ or 10¢ per single box. How much would the merchant gain by selling the entire case by the single box?

6. If 7 gal. of molasses cost \$3.75, what will 22 gal. cost?

7. 8 books cost \$16. What is the cost of 11 books?

8. If $\frac{1}{4}$ acre of land costs \$$x$, what will $\frac{9}{16}$ acre cost?

9. If $\frac{1}{8}$ lb. of tea costs x¢, what will $\frac{1}{2}\frac{6}{0}$ lb. cost?

10. If $\frac{3}{4}$ yd. of ribbon costs b¢, what will $\frac{5}{8}$ yd. cost?

11. If x bbl. of flour can be bought for .06 of your money, how many barrels can be bought for .6 of your money?

12. If a yard of cloth can be bought for \$$\frac{5}{6}$, what part of a yard can be bought for \$$\frac{5}{12}$? for \$$\frac{5}{24}$?

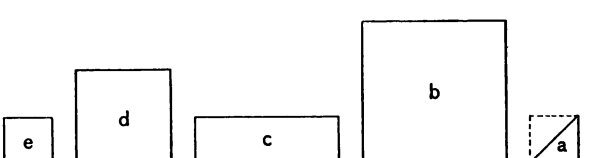
13. If $\frac{5}{8}$ bu. of seed can be bought for \$.60, how many $\frac{5}{8}$ bu. can be bought for $2\frac{3}{8} \cdot 60$¢? how many bushels?

14. $\frac{3}{5}$ lb. of tea costs \$.40. What is the cost of a pound?

15. If it costs \$1 to saw a cord of wood so as to make 3 parts of each stick, what will it cost to saw it so as to make 4 parts of each stick?

16. A pole x ft. long casts a 5-ft. shadow. What is the length of a pole which casts a shadow of 30 ft. at the same time of day?

17. If a 15-ft. staff casts a shadow of 10 ft., what is the length of a staff which casts a shadow of 70 ft. at the same time of day?



Relations of Squares.— 1. Draw these diagrams on the blackboard, making e a foot long.

2. If e is a foot long, what is the length and height of each of the others? Express the dimensions of each in yards or parts of a yard.

3. Discover all the ratios that you can.

4. 1 is the ratio of e to what part of c? to what part of d? of b? of the triangle a?

5. What is the ratio of d to each of the others? of c? of b? of the triangle a?

6. If the length of e is 1, what is the length of d? of b?

7. If e is the 1^2 (read, If e is the square of 1), d is the square of what? b is the square of what?

8. 1 is the ratio of 1^2 to what part of 2^2? What is the ratio of the 1^2 to the 3^2?

9. What is the ratio of the 2^2 to each of the other squares?

10. What is the ratio of the 3^2 to each of the other squares?

11. What is the least number of equal squares into which e can be separated? What is the ratio of one of these squares to e? to each of the other rectangles?

12. What is the least number of squares into which b can be separated? What is the ratio of e to any of these squares?

13. What is the ratio of $\frac{1}{4}$ of b to each of the other figures?

14. Into how many 4-in. sq. can e be separated?

15. What is the ratio of a 4-in. sq. to each of the other figures?

16. Into how many 3-in. sq. can e be separated?

17. Into how many 2-in. sq. can e be separated?

18. Into how many rectangles 1 ft. long and 1 in. high can e be separated?

19. The ratio of d to b equals the ratio of e to what part of b?

20. What is the ratio of a square foot to a square inch? of a square yard to a square foot? of a square yard to a 2-ft. sq.? of a square yard to a 3-ft. sq.? to 9 sq. ft.? of a square foot to a 6-in. sq.? of a 6-in. sq. to a square inch?

Mensuration — Estimating. — 1. Draw a rectangle. Estimate the length of the edges. Measure.

2. Draw rectangles 1 by 2, 1 by 3, 2 by 3, etc.

3. Practice estimating the length of lines drawn on the blackboard. Express in feet and parts of feet.

4. Try to draw lines of a given length.

Ex.: Draw a line $2\frac{1}{2}$ ft. long. Measure. Practice.

5. Place dots on the blackboard and estimate distance apart. Measure. Practice.

6. Try to draw lines of a given length with eyes closed. Draw the lines in different positions.

Make a diagram similar to the following, large enough to contain ten estimates, measures, and differences. Fill out the diagram. Do not try in this exercise to estimate the length of long lines or edges.

OBJECT.	DIMENSION.	ESTIMATE.	MEASURE.	DIFFERENCE.
Table.	Width.	2 ft. 8 in. $2\frac{2}{3}$ ft.	2 ft. 6 in. $2\frac{1}{2}$ ft.	2 in. $\frac{1}{6}$ ft.

Make a diagram similar to the following. Make ten estimates in feet of the length of lines. Measure. Write difference between estimate and measure. Express the length of the line in yards. One example is given below.

OBJECT.	DIMENSION.	ESTIMATE.	MEASURE.	DIFFERENCE.	YARDS.
Blackboard.	Length.	19 ft.	$17\frac{3}{4}$ ft.	$1\frac{1}{4}$ ft.	$5\frac{1}{2}\frac{1}{2}$ yd.

19 ft. $17\frac{3}{4}$ ft. and what equal 18 ft.
$17\frac{3}{4}$ ft. 18 ft. and what equal 19 ft.?
$1\frac{1}{4}$ ft. Then $17\frac{3}{4}$ ft. and what equal 19 ft.?

Make another diagram. Make ten estimates in yards of the length of lines. Measure. Write differences. Express length in feet.

OBJECT.	DIMENSION.	ESTIMATE.	MEASURE.	DIFFERENCE.	FEET.
Room.	Width.	10 yd.	$10\frac{2}{3}$ yd.	$\frac{2}{3}$ yd.	32 ft.

$10\frac{2}{3}$
$\frac{3}{32}$

What is the *number* of yards in the width of the room?

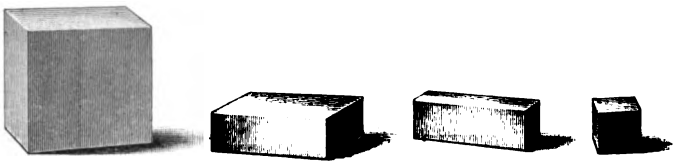
The number of feet equals how many times the number of yards?

$3 \cdot 10\frac{2}{3}$ equals what?

What, then, is the number of feet?

Try to draw a square yard. Observe and tell what you think is wrong. Measure.

Try to draw lines 1 yd. long on the board. Observe carefully and tell whether you think the lines more, less, or a yard long. Draw them in different positions.



Place a cube 1 in. long, another 3 in. long, a solid 3 in. by 1 in. by 1 in., and another 3 in. by 3 in. by 1 in. where they can be observed.

Ratios. — 1. Tell all you can about the ratios of these units.

2. Into how many *c*'s can *d* be cut? into how many *b*'s? into how many *a*'s?

3. What is the ratio of *d* to each of the other solids? of *c* to each? of *b* to each? of *a* to each?

4. If *c* is 1, what is the name of each of the other units?

5. If *b* is 1, what is the name of each of the other units?

6. If *d* is 1, what is the name of each?

7. The sum of *b* and *c* equals what part of *d*?

8. What is the ratio of the sum of *b* and *c* to *a*? of *d* to the sum of the other three?

9. If the edge of *a* is 1, what is an edge of *d*?

10. If *a* is the cube of 1, *d* is the cube of what?

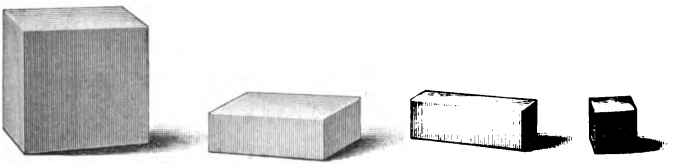
11. What is the ratio of the cube of 1 to the cube of 3? of the cube of 3 to the cube of 1?

12. What is the least number of equal cubes into which *a* can be cut? into which *d* can be cut?

13. What is the ratio of $\frac{1}{3}$ of *a* to $\frac{1}{3}$ of *d*?

What is the ratio of 2 *a*'s to 2 *d*'s? of $\frac{1}{2}$ of *a* to $\frac{1}{2}$ of *d*? of $\frac{1}{2}$ of *d* to $\frac{1}{2}$ of *a*?

14. Into how many cubes $\frac{1}{3}$ in. long can each solid be cut?



Ratios — Imaging. — 1. Draw these prisms on the blackboard, making *a* 1 ft. long.

2. Write all the ratios that you can.

3. If *a* is 1 ft. long, what are the dimensions of the other prisms?

4. Express the dimensions of each prism in yards or parts of yards.

Ex.: The prism *b* is 1 yd. by $\frac{1}{3}$ yd. by $\frac{1}{3}$ yd.

5. If *a* is a cubic foot, *d* is what?

6. What is the ratio of the cubic yard to each of the other prisms?

7. What is the ratio of a cube 2 ft. long to each of these prisms? What is the ratio of each of the prisms to a 2-ft. cube?

8. If there is no waste in cutting, into how many cubic feet can a cubic yard be cut?

9. How many cubic feet in 1 layer of the cubic yard? How many cubic feet in 1 row of 1 layer of the cubic yard?

10. Show the position of the middle cube of the front row of the bottom layer.

How many cubic feet have surfaces touching this cube? How many touch it by edges? how many by points only?

11. In what row and in what layer is the middle cube of the cubic yard?

How many cubes touch the middle cube by a surface? how many by edges only?

How many cubes of the cubic yard touch the middle cube of the cubic yard?

12. How many square yards in the lateral surface of the cubic yard? how many square feet?

13. How many of the cubic feet in the cubic yard have but one surface forming part of the surface of the cubic yard? how many have two surfaces? how many have three?

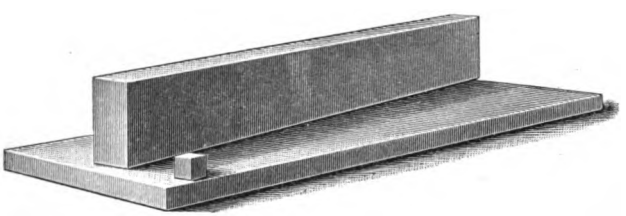
14. How many cubic yards can be placed against the walls of a room 9 ft. in each dimension?

15. What are the dimensions of the unoccupied space?

16. A stone 6 ft. by 3 ft. by 3 ft. contains how many cubic yards?

17. A beam of wood 60 ft. long, 3 ft. thick, and 3 ft. wide contains how many cubic yards?

18. How many loads of a cubic yard each will be removed in making an excavation 27 ft. long, 15 ft. wide, and 12 ft. deep?



Board Measure. — 1. Bring to the class a board 1′ by 1′ by 3′ or 4′ and a piece of 2″ by 4″ studding.

A board foot is 1′ by 1′ by 1″.

2. How many board feet in the board?

3. The piece of studding equals how many board feet?

4. A cubic foot equals how many board feet?

5. What is the length of a board 1″ thick and 1′ wide equivalent to a cubic foot?

6. A stick of timber 1′ by 1′ by 12′ equals how many board feet?

Lumber an inch thick or less is sold by the square foot. A square foot of such lumber is a board foot. Lumber more than an inch thick is computed in board feet at its actual thickness.

A board having a surface of 12 sq. ft. and less than 1 in. thick is sold as 12 board feet.

A plank having a surface of 12 sq. ft. and $3\frac{1}{2}$ in. thick is sold as 42 board feet.

7. How many board feet in 3 boards 1″ by 1′ by 13′?

8. In a 2″ plank 14′ long and $1\frac{1}{2}$′ wide, how many board feet?

9. How many feet of lumber in 3 pieces of 2″ by 4″ studding, each 9′ long?

10. How many feet of lumber in 5 pieces of 2″ by 4″ studding, each 15′ long?

11. How many feet of lumber in a board

12 ft. long, 1 ft. wide, and 1 in. thick?
36 " " 1 " " " 1 " "
6 " " 1 " " " 1 " "
6 " " 2 " " " 1 " "
24 " " 1 " " " 2 " "

12. Find the number of board feet in

6 planks 9 ft. by 1 ft. by 1 in.
12 pieces scantling 12 ft. by 2 in. by 2 in.
8 joists 14 ft. by 2 in. by 2 in.

13. How much inch lumber is needed to lay a floor 16 ft. by 24 ft.?

14. How many feet of lumber in 30 pieces of siding, each 16 ft. long, 6 in. wide, and $\frac{1}{2}$ in. thick?

15. How much is the cost of 10 planks 12 ft. long, 1 ft. wide, and 2 in. thick at $12 a thousand?

16. Find the cost of the following bill at \$14 a thousand:

6 beams 24 ft. by 10 in. by 10 in.
18 scantlings 16 ft. by 4 in. by 2 in.
100 boards 16 ft. by 6 in. by 1 in.
50 planks 12 ft. by 1 ft. by 2 in.

17. How many feet of lumber $1\frac{1}{2}$ in. thick are needed for a platform 40 ft. long and 24 ft. wide?

Relations of Solids. — 1. What is the number of cubic inches in a solid 1″ by 1″ by 2″? in a solid 1″ by 1″ by 3″? in a solid 1″ by 1″ by $1\frac{1}{2}$″? in a solid 1″ by 1″ by $3\frac{1}{3}$″? in a solid 2″ by 1″ by $3\frac{1}{3}$″?

2. What is the ratio of a solid 2 by a by b to a solid 4 by a by b?

3. What is the ratio of a solid 3 by a by b to a solid 1 by a by b?

4. What is the ratio of a solid $\frac{1}{4}$ by a by b to a solid 1 by a by b?

5. What is the ratio of a solid $\frac{1}{2}$ by a by b to a solid 2 by a by b?

1. What is the ratio of a solid 2 by 3 by 5 to a solid 4 by 3 by 5?

2. What is the ratio of a solid 3 by a by b to a solid 1 by a by b?

3. What is the ratio of a solid 1 by a by b to a solid $\frac{1}{2}$ by a by b?

4. What is the ratio of a solid $\frac{5}{4}$ by a by b to a solid 1 by a by b?

5. What is the ratio of a solid a by b by c to a solid $\frac{a}{2}$ by b by c?

6. What is the ratio of a solid $\frac{a}{3}$ by a by a to a solid $\frac{a}{2}$ by a by a?

Relations of Rectangles. — 1. What is the number of square inches in a rectangle 2″ by 3″? 1″ by 2″? 1″ by $2\frac{1}{2}$″? 1″ by $3\frac{1}{2}$″? $1\frac{1}{2}$″ by 1″? 1″ by $5\frac{1}{3}$″? 2″ by $1\frac{1}{2}$″? 1″ by $2\frac{2}{3}$″? 1″ by $3\frac{3}{4}$″?

2. If 3 is the number of square inches in a rectangle 3″ long, what is its altitude?

3. If 1 is the number of square inches in a rectangle 2″ long, what is its altitude?

If 2 is the number of square inches in a rectangle 4″ long, what is its altitude?

4. If 1 is the number of square inches in a rectangle 3″ long, what is its altitude?

5. If 6 is the number of square inches in a rectangle 2″ long, what is its altitude?

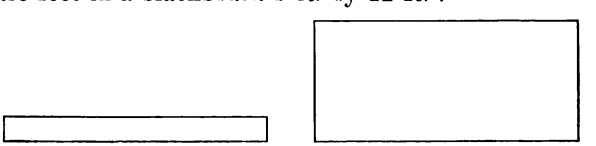
6. If 1 is the number of square feet in a rectangle 2′ long, what is its altitude?

7. If 1 is the number of square inches in a rectangle 7″ long, what is its altitude? If 7 were the number of square inches in the rectangle 7″ long, what would be its altitude? What, then, is its altitude if 1 is the number?

8. If $2\frac{1}{2}$ is the number of square inches in a rectangle $2\frac{1}{2}$″ long, what is its altitude? If 1 is the number of square inches in the rectangle $2\frac{1}{2}$″ long, what is its altitude?

9. Two rectangles are equal in length. The area of one is twice the area of the other. What is the ratio of their altitudes?

Mensuration of Surfaces. — 1. What is the number of square feet in a blackboard 5 ft. by 11 ft.?



What is the number of square feet in a blackboard 1 ft. by 11 ft.?

What, then, equals the number in a blackboard 5 ft. by 11 ft.?

Ans.: 11 is the number of square feet in a blackboard 1 ft. by 11 ft.

$\therefore 5 \cdot 11$ equals the number in a blackboard 5 ft. by 11 ft., and 55 is the number.

What is the number of square feet in $\frac{1}{5}$ of a blackboard 5 ft. by 11 ft.?

What, then, equals the number in the entire blackboard?

2. Write and answer many questions similar to problem 1.

3. What is the number of square inches in a 9-in. sq., or in a square 9 in. long?

What is the number of square inches in 1 row?

What, then, equals the number in the square?

What, then, is the number?

Ans.: 9 is the number of square inches in 1 row.

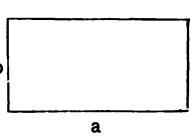
$\therefore 9 \cdot 9$, or 9^2 (read, or the square of 9), equals the number of square inches in the square, and 81 is the number.

9 is the number in $\frac{1}{9}$ of the square.

$\therefore 9^2$ equals the number in the square.

4. Write answers to many questions similar to 3.

5. If a is the number of units in the length of a rectangle and b the number in its width, what equals the number in its area? ab equals number in its area.



If b is the number of units in the length of a square, what equals the number in its area?

6. If 55 is the number of square feet in the area of a blackboard 11′ long, what is its altitude?

If 11 is the number of square feet in a blackboard 11′ long, what is its altitude?

What, then, is its altitude if 55 is the number?

$\frac{55}{11}$ = altitude.

What ratios are equal?

What is the ratio of the areas of the rectangles? of the altitudes? of the lengths?

What is the ratio of 5′ to 1′? of 55 to 11?

7. Write answers to questions similar to question 6.

8. What is the number of square feet in a rectangle 3′ long and $2\frac{1}{2}'$ high?

What is the number if the rectangle is only 1′ high?

What, then, equals the number if it is only $\frac{1}{2}'$ high?

What, then, equals the number if it is $2\frac{1}{2}'$, or $\frac{5}{2}'$, high?

What, then, is the number?

$$\frac{5 \cdot 3}{2} = \text{number.}$$

9. Write and answer questions similar to 8.

10. What is the number of units in a rectangle $3\frac{1}{4}'$ long and $2\frac{2}{3}'$ high?

What is the number if the rectangle is only 1′ high?

What, then, equals the number if it is only $\frac{1}{3}'$ high?

What, then, equals the number if it is $2\frac{2}{3}'$, or $\frac{8}{3}'$, high?

What is the number?

$$\frac{\overset{2}{\cancel{8}} \cdot 13}{3 \cdot \cancel{4}} = \text{number.}$$

11. What is the number of units in a rectangle $15\frac{2}{3}'$ long and $7\frac{3}{5}'$ high?

$$\frac{38 \cdot 47}{5 \cdot 3} = \text{number.}$$

12. Write and answer many questions similar to 11.

13. If 17 is the number of square inches in a rectangle 3″ long, what is its altitude?

If 1 were the number of square inches in the area, what, then, would be the altitude?

What, then, is the altitude if 17 is the number.

$$\frac{17}{3} = \text{altitude.}$$

14. Write and answer similar questions.

15. If $18\frac{3}{4}$ is the number of units in the area of a rectangle $5\frac{1}{2}'$ long, what is its altitude?

If $5\frac{1}{2}$ were the number in the area, what would be the altitude?

If 1 were the number, what would be the altitude?

$\frac{75 \cdot 2}{4 \cdot 11}$ = altitude.

If $\frac{1}{4}$ were the number, what would be the altitude?

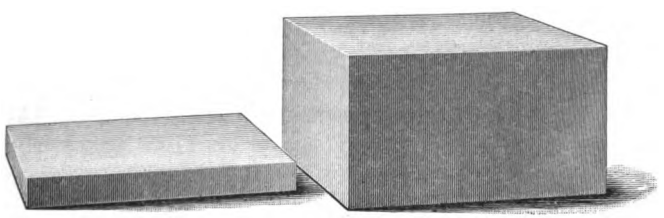
What, then, equals the altitude if the number is $18\frac{3}{4}$, or $\frac{75}{4}$?

What is the altitude?

Ans.: If 1 were the number in the area, $\frac{2}{11}$ would be the altitude. If $\frac{1}{4}$ were the number, $\frac{1}{4}$ of $\frac{2}{11}$ would equal the altitude, and $3\frac{9}{22}$ is the altitude.

Mensuration of Solids. — 1. What is the number of layers 1″ thick in a solid 7″ high?

2. What is the number of cubic inches in a solid 5″ by 8″ by 9″?



What equals the number of cubic inches in a solid 1″ by 8″ by 9″?

5 · 8 · 9's = number.

What, then, equals the number in a solid 5″ by 8″ by 9″?

What is the number?

Ans.: 8 · 9's equals the number in a solid 1″ by 8″ by 9″.

$\therefore$ 5 · 8 · 9's equals the number in a solid 5″ by 8″ by 9″, and 360 is the number.

3. Write answers to many similar questions.

4. What is the number of cubic feet in a cube 7′ long?

How many layers 1′ thick in the cube?

$7 \cdot 7 \cdot 7$, or 7^3 = number.

What equals the number of cubic feet in 1 layer? in the cube?

What is the number?

Ans.: $7 \cdot 7$, or 7^2, equals the number of cubic feet in 1 layer.

$\therefore$ $7 \cdot 7 \cdot 7$, or 7^3, equals the number in the cube.

5. Write answers to similar questions.

1. What part of 8 · 9's equals the number of cubic feet in a solid $\frac{1}{2}'$ by 8′ by 9′?

$$\frac{8 \cdot 9}{2} = \text{number.}$$

2. What part of 8 · 9's equals the number of cubic feet in a solid $\frac{2}{3}'$ by 8′ by 9′? What is the number?

3. What is the number of cubic yards in a solid $2\frac{1}{2}$ yd. by 7 yd. by 11 yd.?

What equals the number if the solid is only 1 yd. high?

$\frac{5 \cdot 7 \cdot 11}{2}$ = number.

What part of 7 · 11's equals the number if the solid is only $\frac{1}{2}$ yd. high?

What, then, equals the number if the solid is $2\frac{1}{2}$, or $\frac{5}{2}$ yd., high?

What is the number?

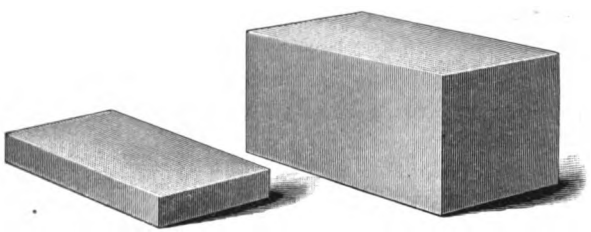
4. Write answers to similar questions.

5. Make sentences like this: $\frac{1}{2}\frac{1}{7}$ of 8 · 15 · 32's equals the number of cubic yards in an excavation, if 8 · 15 · 32's equals the number of cubic feet in it.

1. If 6 is the number of cubic inches in a solid 2″ by 3″, what is its altitude? If 3 is the number of cubic inches, what is its altitude? If 1 is the number, what is its altitude?

2. If 12 is the number of cubic inches in the volume of a solid 3″ by 4″, what is its altitude? if 6 is the number? if 1 is the number?

3. If 180 is the number of cubic inches in a solid 4″ by 9″, what is its altitude?



If 36 is the number of cubic inches in the solid, what is its altitude?

What, then, is its altitude if 180 is the number?

$\frac{180}{36}$ = altitude.

If their bases are equal, the altitude of a solid of 180 cu. in. is how many times as great as the altitude of a solid of 36 cu. in.?

4. If 180 is the number of cubic inches in a solid 4″ by 9″, what is its altitude?

If 1 were the number of cubic inches in a solid 4″ by 9″, what would be its altitude?

$\frac{180}{36}$ = altitude.

What, then, is the altitude if 180 is the number?

Ans.: $\frac{1}{36}$ of an inch is the altitude if 1 is the number.

$\therefore \frac{180}{36}$ in., or 5 in., is the altitude if 180 is the number.

5. Write answers to similar questions.

6. What is the length of a cord of 2-ft. wood piled 5 ft. high?

7. A cord of 27-in. wood is 12 ft. long. How high is it?

8. There are 90 cords of wood in a range 10 ft. wide and 6 ft. high. What is the length of the range?

9. What is the number of cubic feet in a solid $3\frac{1}{2}'$ by $3'$ by $4'$?

$\frac{7 \cdot 3 \cdot 4}{2}$ = number.

What equals the number if the solid is only $1'$ high? if it is only $\frac{1}{2}'$ high? if it is $3\frac{1}{2}'$, or $\frac{7}{2}'$, high?

Ans.: 3 4's equals the number if the solid is $1'$ high, $\frac{1}{2}$ of $3 \cdot 4$'s equals the number if the solid is $\frac{1}{2}'$ high, and $\frac{7}{2}$ of $3 \cdot 4$'s equals the number if it is $3\frac{1}{2}'$, or $\frac{7}{2}'$, high.

10. Write answers to similar questions.

11. How many cubic feet in a bin $5'$ by $2\frac{3}{4}'$ by $7'$?

$\frac{5 \cdot 11 \cdot 7}{4}$ = number.

If the bin were $1'$ by $1'$ by $7'$, how much would it hold?

If it were $1'$ by $\frac{1}{4}'$ by $7'$, what would it hold?

If it were $1'$ by $\frac{11}{4}'$ by $7'$, what would it hold?

What, then, is the number of cubic feet since it is $5'$ by $\frac{11}{4}'$ by $7'$?

12. What is the number of bushels in a bin $7\frac{1}{2}'$ by $4\frac{2}{3}'$ by $12\frac{3}{4}'$?

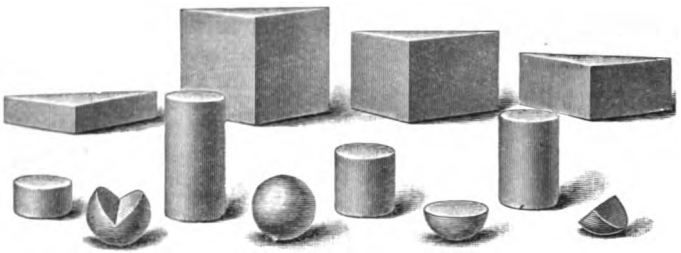
$\frac{4 \cdot 15 \cdot 14 \cdot 51}{5 \cdot 2 \cdot 3 \cdot 4}$ = number.

$\frac{15}{2}$ of $\frac{14}{3}$ of $\frac{51}{4}$ equals the number of what?

$\frac{4}{5}$ of $\frac{15}{2}$ of $\frac{14}{3}$ of $\frac{51}{4}$ equals the number of what?

13. What equals the number of pecks in a bin 8 ft. by $3\frac{1}{2}$ ft. by $6\frac{2}{3}$ ft.?

What is the number?



Ratios expressed in Per Cent. — 1. If *a* represents 100% (100 per cent), each of the other units represents how many per cent?

2. Compare 25% with 50%, with 75%, and with 100%.

3. Compare 50% with each of the others; compare 75% with each; compare 100% with each.

4. Draw rectangles representing 100%, 75%, 50%, and 25%.

Compare the units drawn, using the per cent names.

5. Select solids representing these different per cents, and compare.

6. Show me 50% of the blackboard; of different solids, surfaces, and lines.

7. If you call *c* 100%, what is each of the others? if you call *d* 100%?

8. Show me 100%, 75%, 50%, and 25% of the different units.

9. Make sentences like this: If 6 equals 100%, 3 equals 50%.

10. Make sentences like this: If 5 equals 25%, 4 · 5's equals 100%.

11. What is the ratio of 100% to 75%? of 75% to 50%? of 50% to 75%? to 100%?

12. What is the ratio of $\frac{1}{2}$ of 50% to 25%? to 100%? to $\frac{1}{3}$ of 75%?

13. What is the ratio of $\frac{1}{3}$ of 75% to $\frac{1}{4}$ of 100%? to 25%? to 50%?

14. Review without the diagrams or solids.

Problems. — 1. At 25¢ a peck, how many pecks of potatoes can be bought for \$1? for \$5? for \$7? for \$.75?

2. \$.20 is the cost of a ball. The ratio of the selling price to the cost is $\frac{5}{4}$. What is the selling price?

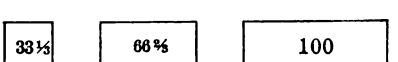
3. 28 equals 100% of the cost of a cow. The selling price equals 125% of the cost. What is the selling price?

4. A boy sells a knife at a gain of 50%. The gain equals what part of the cost? What is the ratio of the selling price to the cost?

5. A boy bought a sled for \$4 and sold it for \$5. What per cent of the cost did he gain?

6. A man bought a lot for \$200 and sold it at a gain of 50%. For how much did he sell it?

7. If oranges are worth x¢ a dozen and Fred buys them for 75% of their value, what part of x¢ does he pay for them?



Relations. — 1. Draw units having the ratio of $33\frac{1}{3}$, $66\frac{2}{3}$, and 100.

2. If the smallest unit is $33\frac{1}{3}$, each of the others equals how many $33\frac{1}{3}$?

3. What is the ratio of $33\frac{1}{3}$ to each of the others? of $66\frac{2}{3}$ to each? of 100 to each?

4. Find solids, surfaces, and lines representing 100, $66\frac{2}{3}$, $33\frac{1}{3}$.

5. Draw the units to represent 100, $66\frac{2}{3}$, and $33\frac{1}{3}$.

6. If you pay $33\frac{1}{3}$¢ for 5, how many 5's can you buy for $66\frac{2}{3}$¢? for \$1?

7. At $33\frac{1}{3}¢$ a yard, how many yards of carpet can you buy for \$1? for \$5? for \$7? for \$4? for \$9? for \$10?

8. If you can buy x lb. of coal for $66\frac{2}{3}¢$, how much can you buy for $33\frac{1}{3}¢$? for \$1? for \$6?

9. What is the ratio of 100 to $33\frac{1}{3}$? of [illegible] 0? of 400? of 700? of 600? of 1200?

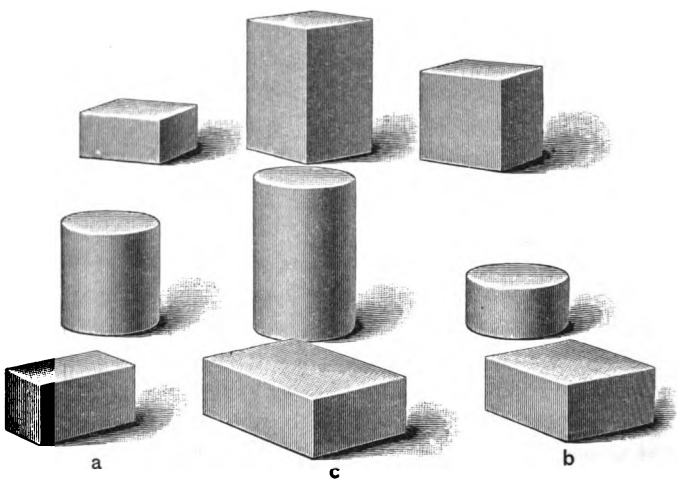
10. What is the ratio of 100 to $66\frac{2}{3}$?

What, then, is the ratio of 600 to $66\frac{2}{3}$? of 500? of 700? of 900?

Draw 3 lines a, b, c, so that the ratio of c to b is 3, of b to a, 2.

11. Tell what you can of the relations of these lines.

What is the relation of c to a? of a to c? of a to $\frac{1}{3}$ of c? of a to b?



Ratios. — 1. If c represents 100%, what part of 100% is represented by a? by b?

2. Tell all you can about the relation of these units.

3. Compare $33\frac{1}{3}$% with each of the others.

4. Compare $66\frac{2}{3}\%$ with each; compare 100% with each.

5. Show me 100% of some object in the room; $33\frac{1}{3}\%$ of it; $66\frac{2}{3}\%$ of it.

6. Select sets of solids that represent 100%, $66\frac{2}{3}\%$, and $33\frac{1}{3}\%$.

7. Draw rectangles representing 100%, $66\frac{2}{3}\%$, and $33\frac{1}{3}\%$.

What other names could you give to rectangles having the same ratios?

8. Make sentences like this: 50% of 6 is 3; 50% of 11 is $5\frac{1}{2}$.

9. Make sentences like this: If 9 apples equal 100%, 3 apples equal $33\frac{1}{3}\%$.

10. Make sentences like this: If 14 equals $66\frac{2}{3}\%$, $\frac{1}{2}$ of 14 equals $33\frac{1}{3}\%$, and $\frac{3 \cdot 14}{2}$ equals 100%.

11. Show per cents of different units.

Ex.: These 2 rows of seats are 25% of the 8 rows.

12. What is the ratio of 100% to $33\frac{1}{3}\%$?

\$40 equals $33\frac{1}{3}\%$ of the cost of a cow. What equals its cost?

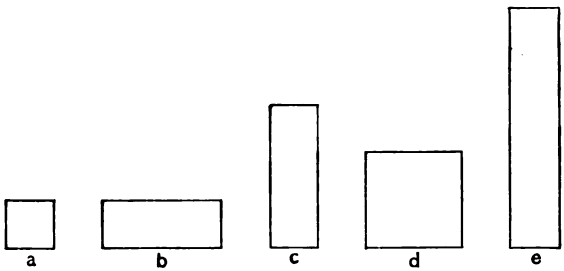
13. If goods bought at $\$x$ were sold at a gain of $33\frac{1}{3}\%$, the selling price equals what per cent of the cost?

14. Bought a book for \$3 and sold it for \$2. What per cent did I lose?

15. A merchant sells goods for \$1 that cost \$.75. The gain equals what part of the cost? What per cent?

16. A hat cost \$6 and was sold for \$4. What per cent was lost? What was the ratio of the selling price to the cost?

17. A hat cost \$4 and was sold for \$6. What per cent was gained? What was the ratio of the selling price to the cost?



Ratios. — 1. Select solids that have the ratio of 1, 2, 3, 4, 5.

If the largest is 1, each of the others equals what part of 1?

2. Compare each with the other four.

3. If the largest unit is 100, each of the others equals what part of 100?

4. Show me the 60; the 80; the 40.

5. Compare each with the other four.

6. Study these figures until you are familiar with their ratios.

7. Make the comparisons without the solids.

8. What is the ratio of 100 to 20? of 200? of 500? of 400? of 600? of 800? of 700?

9. At \$20 an acre, how many acres of land can be bought for \$100? How many acres can be bought for \$300? for \$700? for \$400? for \$900? for \$600?

10. What is the ratio of 100 to 40? 300 equals how many $\frac{5 \cdot 40}{2}$? 400? 600? 700? 800? 900?

11. Review until the answers can be given instantly.

1. Find all the ratios you can.

Ex.: I see the ratio 3.

2. If the largest unit is 100%, each of the others equals what part of 100% ?

3. If a is 20%, each of the others is how many per cent ?

4. Compare each per cent with the other four.

5. Select solids having the same relations and compare the per cents.

6. Find surfaces representing the same relations and express relations in per cent.

7. Draw lines having the same ratios. Call the longest line 100% and compare.

8. Show me 20% of the door. Show me 20% of other objects. Find 40%.

9. What is the ratio of e to d ?

If we call d 100%, what is each of the others ?

10. What is the ratio of e to c ?

If we call c 100%, what is each of the others ?

11. Lucy had 15 pears. She gave away 3. What per cent did she give away ? What per cent did she keep ?

12. A watch costing \$100 was sold for \$80. The loss equaled what per cent of the cost ?

If a watch costs \$80 and sells for \$100, what is the ratio of the selling price to the cost ?

1. Select solids that are to each other as 1, 2, 3, 4, 5, 6.

Tell the ratios that you see.

In this set of solids, can you find the ratio 2 repeated three times ?

2. If the largest unit is 1, each of the other units equals what part of 1 ?

3. Tell all the ratios that you see.

Ex.: 2 is the ratio of $\frac{1}{3}$ to $\frac{1}{6}$.

Tell ratios again, using the names $16\frac{2}{3}$, etc.

4. If the largest unit is 100, each of the other units equals what part of 100 ?

5. If the smallest unit is $16\frac{2}{3}$, each of the others equals how many $16\frac{2}{3}$?

6. Point to different units and tell how many $16\frac{2}{3}$ there are in each ?

7. Draw a line and call it $16\frac{2}{3}$. Draw a line 6 times as long. What is the name of this line ?

8. What part of this line equals $83\frac{1}{3}$?

9. $16\frac{2}{3}$ $16\frac{2}{3}$ $16\frac{2}{3}$ $16\frac{2}{3}$ $16\frac{2}{3}$ $16\frac{2}{3}$ Discover as many relations as you can between the different numbers of $16\frac{2}{3}$. What is the sum of $6 \cdot 16\frac{2}{3}$? Practice adding to 100 by $16\frac{2}{3}$. How many $16\frac{2}{3}$ in $\frac{1}{2}$ of 100 ? in $\frac{2}{3}$ of 100 ? in $\frac{5}{6}$ of 100 ? in $\frac{5}{6}$ of 100 ? in $33\frac{1}{3}$? in 50 ? in 100 ? in $66\frac{2}{3}$? in $83\frac{1}{3}$?

10. Draw a rectangle whose ratio to $\frac{1}{2}$ of $66\frac{2}{3}$ is 3.

11. Make sentences like the following: $83\frac{1}{3}$ equals $\frac{5}{6}$ of 100 ; $\frac{5}{4}$ of $66\frac{2}{3}$; $\frac{5}{3}$ of 50 ; $\frac{5}{2}$ of $33\frac{1}{3}$; and 5 times $16\frac{2}{3}$.

12. Make sentences like the following: 5 is the ratio of $83\frac{1}{3}$ to $\frac{1}{6}$ of 100 ; to $\frac{1}{4}$ of $66\frac{2}{3}$; to $\frac{1}{3}$ of 50 ; to $\frac{1}{2}$ of $33\frac{1}{3}$; and to $16\frac{2}{3}$.

13. What is the ratio of 100 to $16\frac{2}{3}$?

What, then, is the ratio of 300 to $16\frac{2}{3}$? of 500 ? of 700 ? of 400 ? of 50 ?

14. At $16\frac{2}{3}$¢ a yard, how many yards of gingham can be bought for \$1 ? for \$3 ? for \$5 ? for \$7 ?

15. What is the ratio of the cost of x yd. of ribbon at $66\frac{2}{3}$¢ a yard to the cost at $16\frac{2}{3}$¢ a yard ?

Ratios and Problems. — 1. Select a different set of solids having the ratios of those above.

2. If we call the smallest $16\frac{2}{3}\%$, what is the name of each of the other units ?

3. Compare $16\frac{2}{3}\%$ with each of the others.

4. Tell all the ratios that you can.

Ex.: 3 is the ratio of 100% to $\frac{3}{5}$ of 50%.

5. Make sentences like this: 2 equals $16\frac{2}{3}\%$ of 12; 5 equals 50% of 10.

What is the ratio of 100% to $16\frac{2}{3}\%$? of 500% to $16\frac{2}{3}\%$? of 400%? of 700%? of $33\frac{1}{3}\%$? of $83\frac{1}{3}\%$? of 50%? of $66\frac{2}{3}\%$?

6. If a man buys for \$6 and sells for \$5, his loss equals what per cent of the money invested?

7. If he buys for \$1 and sells for $83\frac{1}{3}¢$, the loss equals what per cent of the cost?

8. What per cent is gained by buying apples at $20¢$ a peck and selling them for $25¢$?

9. Fred has $75¢$ and loses $25¢$. What per cent of his money does he lose?

10. $\frac{1}{8}$ of the pupils in a school were absent. What per cent were present?

11. What is the ratio of the entire number of pupils to the number present?

Problems. — 1. What is the ratio of the number of *a*'s in 100 to the number in 25? to the number in $33\frac{1}{3}$? in 50? in $66\frac{2}{3}$? in $12\frac{1}{2}$? in 75?

2. At $12\frac{1}{2}¢$ a yard, how many yards of ribbon can be bought for \$1? for \$5? for \$7? for \$9?

3. At $33\frac{1}{3}¢$ a pound, how many pounds of coffee can be bought for \$1? for \$4? for \$8?

4. At $16\frac{2}{3}¢$ a pound, how many pounds can be bought for \$5? for \$7?

5. At \$1 a yard, $\frac{1}{8}$ of a yard of cloth costs how many cents? $\frac{2}{3}$ yd.? $\frac{5}{6}$ yd.?

6. What is the cost of 12 boxes of rubber bands at \$1 a box?

7. What is the cost of 40 lb. of cotton at \$1 a pound? at 50¢? at $33\frac{1}{3}$¢? at $12\frac{1}{2}$¢? at $16\frac{2}{3}$¢?

8. What is the cost of 36 lb. of coffee at $16\frac{2}{3}$¢ a pound?

9. At \$1 a bushel, what part of a bushel of wheat can be bought for 50¢? for 25¢? for $66\frac{2}{3}$¢? for $33\frac{1}{3}$¢? for $83\frac{1}{3}$¢? for 60¢? for 40¢?

10. At 20¢ a quart, what part of \$32 will 32 qt. of berries cost? How much will they cost?

11. What is the cost of 72 lb. of sugar at $12\frac{1}{2}$¢ a pound?

12. What is the cost of 36 yd. of flannel at $33\frac{1}{3}$¢ a yard?

13. At 25¢ a bushel, what is the cost of 38 bu. of potatoes?

14. At \$1 a yard, what part of a yard of cloth can be bought for $37\frac{1}{2}$¢?

At \$1 a yard, how much cloth can be bought for \$1.50? for $\$1.66\frac{2}{3}$? for \$1.75? for \$1.20?

1. A man whose salary was \$2000 a year spent 25% of it. How much did he save?

2. A farmer had 150 bu. of wheat and sold $33\frac{1}{3}$% of it. How much did he sell?

3. Mr. Roberts sold a lot for $\frac{2}{5}$ more than he paid for it. What was the ratio of the selling price to the cost?

4. A lot which cost \$600 was sold for $137\frac{1}{2}$% of its value. For what was it sold?

5. Out of a sack containing a bushel of oats, 4 qt. were taken. What per cent was left?

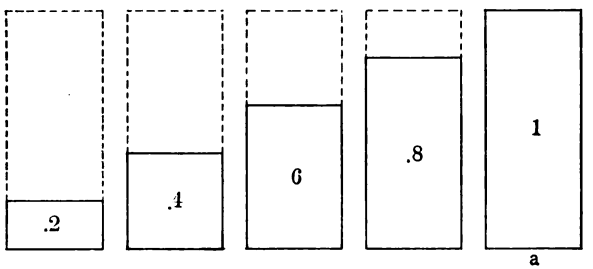
6. 15 bu. equaled 60% of the potatoes a grocer sold. He sold how many thirds of 15 bu.?

7. When hats are bought at \$1 and sold at $\$1.33\frac{1}{3}$, the gain equals what part of the cost? at $\$1.37\frac{1}{2}$? at $\$1.87\frac{1}{2}$?

8. When goods are bought at \$1 and sold at \$.90, the loss equals what per cent of the cost? at 75¢? at 80¢? at $62\frac{1}{2}$¢?

9. What is the cost of 800 bu. of flax at \$1.25 a bushel?

10. What is the cost of 600 lb. of tea at \$1 a pound? at \$1.33⅓ a pound? at \$1.66⅔ a pound? at \$1.75 a pound?



Ratios. — 1. If a is 1, or 1.0 ($1\frac{0}{0}$), what is each of the others?

2. Write the relation of each to each of the others.

Ex.: $.2 = \frac{.4}{2}, \frac{.6}{3}, \frac{.8}{4}$, and $\frac{1}{5}$.

3. Practice until you can think and write these relations readily.

Have pupils associate the names .2, .4, .6, .8, 1 with units having the ratios denoted by these names.

4. What is the ratio of 1 to .6? to 1.1 ($1\frac{1}{0}$)? to .8? to .7? to 1.2?

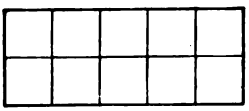
5. What is the ratio of .8 to .6? to 1.1? to .5? to 1.2? to 1.6?

6. Make sentences like this: $\frac{3}{2}$ of \$250 is the cost of .6 of a car of steel rails if .4 of a car of steel rails cost \$250.

$$\frac{3 \cdot \overset{125}{\cancel{250}}}{\cancel{2}} = 375.$$

$\therefore$ \$375 is the cost.

7. What are the names of the equal parts in a?



a

8. If 10 bu. of potatoes cost \$12, what is the cost of 3 bu.?

Ans.: .3 of \$12 equals the cost of 3 bu. of potatoes if 10 bu. cost \$12.

$$3 \cdot 1.2 = 3.6.$$

What is .1 of \$12? What is .3 of \$12?

9. If 10 bu. of potatoes cost \$12, what is the cost of 7 bu.? of 9 bu.? of 13 bu.? of 5 bu.? of 17 bu.? of 15 bu.?

10. If 4 lb. of tea cost \$1.2, what is the cost of 6 lb.?

$$\frac{3 \cdot \overset{.6}{\not 1.\not 2}}{\not 2} = ?$$

11. If 6 lb. of tea cost \$1.8, what is the cost of 4 lb.?

$$\frac{2 \cdot \overset{.6}{\not 1.\not 8}}{\not 3} = ?$$

12. If you can buy 4 lb. of tea for \$1.2, how much can you buy for \$1.8?

$$\frac{\overset{6}{\not 1\not 8} \quad \not 4}{\not 1\not 2\ \not 3} = ?$$

13. If \$1.8 will purchase 6 lb. of tea, what will \$1.2 purchase?

$$\frac{\overset{4}{\not 1\not 2} \quad \not 6}{\not 1\not 8\ \not 3} = 4.$$

14. If 17 yd. of cloth cost \$10, how much can be bought for \$3? for \$7? for \$9? for \$13? for \$11? for \$17?

15. If a staff 20 ft. long casts a 27-ft. shadow, how long a shadow will a 2-ft. staff cast? how long a shadow will a 6-ft. staff cast? how long a shadow will a 14-ft. staff cast? an 18-ft. staff?

Ratios. — 1. Draw a rectangle and separate it into two equal parts.

Separate each half into five equal parts. Separating each half into five equal parts separates the unit into how many equal parts?

2. If you call the entire rectangle 1, each of the other units equals what part of 1?

3. Show me these units: .1, $\frac{1}{5}$, .3, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, .7, $\frac{4}{5}$, and 1.

4. Compare each unit with the other nine.

5. What is the ratio of $\frac{2}{5}$ to .1? to .3? to .7?

6. What is the ratio of .7 to .3? to $\frac{3}{5}$? to $\frac{1}{2}$? to $\frac{4}{5}$? to 1.1?

7. What is the ratio of 1.2 to .7? to .3? to $\frac{4}{5}$? to $\frac{1}{2}$? to $1\frac{1}{2}$?

8. Write sentences like this: If .3 bu. of wheat costs 21¢, $\frac{1}{2}$ bu. will cost $\frac{5}{3}$ of 21¢. Why?

9. Write and answer five questions like this: If .7 yd. of cloth costs 35¢, what is the cost of $\frac{4}{5}$ yd.?

1. $\frac{2}{5}$ is the ratio of what to 1?

$\frac{2}{3}$ is the ratio of what to .6?

2. What is the ratio of .1 to $\frac{1}{2}$ of .1?

What is the ratio of .1 to $\frac{3}{4}$ of .1?

What is the ratio of .1 to $\frac{2}{5}$ of .1?

What is the ratio of $\frac{1}{5}$ to $\frac{3}{4}$ of $\frac{1}{5}$?

3. The ratio of the cost of a ton of hay to the cost of a ton of coal is $\frac{2}{3}$. The cost of the hay is 9. What is the cost of the coal?

4. The ratio of the cost of 7 bbl. of flour to the cost of 7 bbl. of sugar is .4. What is the cost of the flour, if the sugar costs $70?

5. For the use of $420 for one year, a man paid a sum equal to .1 of what he borrowed. How much did he pay?

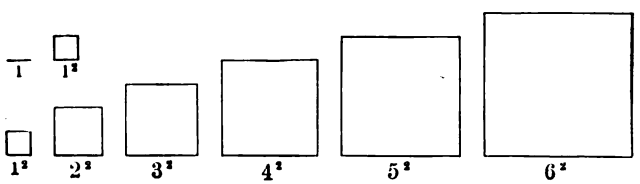
.1 is the ratio of what to $420?

What is the ratio of 42 to 420 ?

6. The ratio of the cost of a hat to the cost of a cape is .4. The cost of the cape is $5. What is the cost of the hat ?

7. If .7 of a field yields 125 bu. of potatoes, how many bushels will the entire field yield ?

8. 12.8 acres of land cost $640. What is the cost of 3.2 acres ?



Ratios of Squares. — 1. Draw a line and call it **1**. Draw 2, 3, 4, 5, 6.

3^2 (read, The square of 3).

2. Draw 1^2, 2^2, 3^2, 4^2, 5^2, 6^2. Show me the 4. Show me 4^2.

3. Study the squares and write the ratios that you see.

4. The 6^2 can be separated into how many 3^2 ?

5. The 4^2 can be separated into how many 2^2 ?

6. The 2^2 can be separated into how many 1^2 ?

7. What is the ratio of 6^2 to 3^2 ? (Read, What is the ratio of the *square of 6* to the *square of 3?*)

8. What is the ratio of 4^2 to 2^2 ? of 2^2 to 1^2 ? of 3^2 to 6^2 ? of 2^2 to 4^2 ? of 1^2 to 2^2 ?

9. The ratio 4 occurs how many times ? the ratio $\frac{1}{4}$? the ratio 9 ? the ratio $\frac{1}{9}$? the ratio $\frac{2}{4}$? the ratio $\frac{4}{9}$?

What other ratios have you found ?

10. What is the ratio of 6^2 to each of the other squares ? of 5^2 ? of 4^2 ? of 3^2 ? of 2^2 ?

11. How many 1^2 in each square? How many 1^2 in the 7^2? 8^2? 9^2? 10^2?

12. Show me the square that equals $16 \cdot 1^2$.

What is the edge of the square that equals $16 \cdot 1^2$?

Ans.: Four is the edge of the square that equals $16 \cdot 1^2$.

13. What is the edge of the square that equals $25 \cdot 1^2$? that equals $9 \cdot 1^2$? that equals $36 \cdot 1^2$? that equals $81 \cdot 1^2$? that equals $64 \cdot 1^2$?

14. What is the edge of the square that equals $4 \cdot 3^2$? that equals $4 \cdot 2^2$? that equals $4 \cdot 1^2$? that equals $4 \cdot 5^2$? that equals $4 \cdot 4^2$?

15. Review without the diagrams again and again.

1. On the blackboard, draw a line and call it 10. Draw $2 \cdot 10$, $3 \cdot 10$, $4 \cdot 10$, $5 \cdot 10$, and $6 \cdot 10$. Draw the square of each.

2. The $(6 \cdot 10)^2$ (read, The square of $6 \cdot 10$'s) can be separated into how many $(3 \cdot 10)^2$?

3. The $(4 \cdot 10)^2$ can be separated into how many $(2 \cdot 10)^2$?

4. The $(2 \cdot 10)^2$ can be separated into how many 10^2?

5. Study the squares and write the ratios that you see.

Ex.: 4 is the ratio of $(6\ 10)^2$ to $(3 \cdot 10)^2$.

6. Make sentences like this: $3 \cdot 10$ is the edge of $(9 \cdot 10)^2$.

7. How many 10^2 in each square?

8. Make sentences like this: $4 \cdot 10$ is the edge of $(16 \cdot 10)^2$.

9. How many 10^2 in $(7 \cdot 10)^2$? in $(8 \cdot 10)^2$? in $(9 \cdot 10)^2$?

10. What is the edge of $(81 \cdot 10)^2$?

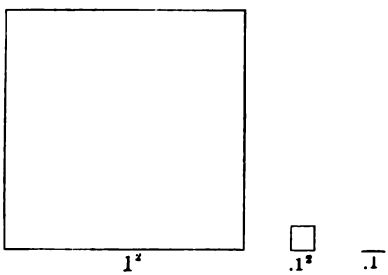
11. Review without the diagrams.

12. The 10^2 equals how many 5^2? 2^2? 1^2? The 8^2 equals how many 4^2? 2^2? 1^2? The 9^2 equals how many 3^2? 1^2?

13. 4 is the ratio of what squares to what other squares?

14. Study the squares and tell how often the different ratios occur.

Ex.: The ratio 4 is found five times.



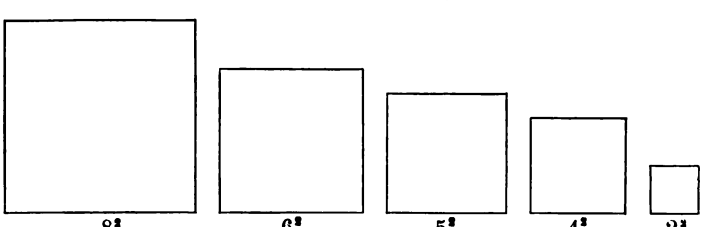
Ratios of Squares expressed in Tenths and Hundredths. — 1. Draw 1. Draw 1^2. Draw .1. Draw $.1^2$.

2. What is the ratio of the .1 to 1? of $.1^2$ to 1^2?

3. Draw .8, .6, .5, .4, .2. Draw the square of each.

4. Make sentences like this: *Four* is the ratio of 1^2 to $.5^2$.

5. In observing the relations of these squares, how many times have you found the ratio 4?



6. What is the ratio of each square to $.2^2$?

7. What is the ratio of each square to $.1^2$?

8. Each square equals how many hundredths of 1^2?

9. To how many hundredths of 1^2 is each of the following equal: $.1^2$? $.2^2$? $.3^2$? $.4^2$? $.5^2$? $.6^2$? $.7^2$? $.8^2$? $.9^2$?

10. To how many hundredths of 10^2 is each of the following equal: 1^2? 2^2? 3^2? 4^2? 5^2? 6^2? 7^2? 8^2? 9^2?

Ratio — Interest. — 1. \$8 in a year yields 48¢. What will \$4 yield in the same time?

If 48¢ is paid for the use of \$8 for 1 yr., what is paid for the use of \$4 for the same time?

2. \$5 in a year yields 35¢. What does \$15 yield in the same time?

3. 6% of a principal is \$27. What is 4% of it? What is 8% of it? 7½% of it?

4. What is the ratio of 5% to 6%? of 3% to 6%? of 8% to 6%? of 9% to 6%? of 5½% to 6%? of 7½% to 6%? of 10% to 6%?

5. If .06 of a principal equals the interest at 6% for 1 yr., what part of the principal equals the interest for 8 mo.? for 6 mo.? for 9 mo.?

6. If .06 of the principal equals the interest at 6% for 1 yr., what part of the principal equals the interest for 2 mo.?

What part of .01 of the principal equals the interest for 1 mo.?

7. If 6% of the principal equals the interest for 1 yr., what per cent of the principal equals the interest for 2 mo.? What part of 1% of the principal equals the interest for 1 mo.?

8. If 6% of \$84 equals the interest for 1 yr., 1% of the principal equals the interest for what time?

If 1% of \$84 equals the interest for 2 mo., what part of 1% of \$84 equals the interest for 1 mo.?

What is 1% of \$84? What equals ½ of 1% of \$84?

$.84 equals the interest for what time at 6% ?

$\frac{\$.84}{2}$ equals the interest for what time at 6% ?

9. What equals the interest of $72 at 6% for 1 mo.?

10. What equals the interest of $1000 at 6% for 1 mo.?

11. What equals the interest of $782 at 6% for 1 mo.?

12. Make many sentences like this: $\frac{\$7.82}{2}$ (read, $\frac{1}{2}$ of $\frac{1}{100}$ of $782) equals the interest of $782 at 6% for 1 mo. What is $\frac{\$7.82}{2}$?

13. What equals the interest of $455 at 6% for 5 mo.?

14. Make sentences like this: $\frac{5 \cdot \$4.55}{2}$ (read, $\frac{5}{2}$ of $\frac{1}{100}$ of $455) equals the interest of $455 for 5 mo. at 6%.

15. What is the interest of:

$86 at 6% for 8 mo.? for 15 mo.?
$182 at 6% for 5 mo.? for 1 yr. 5 mo.?
$200 at 6% for 7 mo.? for 11 mo.?

16. What is the interest of $322 at 6% for 7 mo.? at 8% ? at $7\frac{1}{2}$% ?

$$\frac{7 \cdot \$3.22}{2} = \text{interest at } 6\%.$$

$$\frac{4 \cdot 7 \cdot \$3.22}{3 \cdot 2} = \text{interest at } 8\%.$$

$$\frac{5 \cdot 7 \cdot \$3.22}{4 \cdot 2} = \text{interest at } 7\frac{1}{2}\%.$$

17. What is the interest of $250 for 8 mo. at 6% ? at 8% ? at 7% ?

18. What is the interest of $250 for 7 mo. at 6% ? at 8% ? at 7% ?, at $7\frac{1}{2}$% ?

19. What is the interest of $250 at $7\frac{1}{2}$% for 5 mo.? at 5% ? at $4\frac{1}{2}$% ?

20. What is the interest of \$120 for 7 mo. at 6%? at 5%? at $8\frac{1}{2}$%?

21. What is the interest of \$400 for 2 yr. 9 mo. at 6%? at $6\frac{1}{2}$%? at 5%?

22. What is the interest of \$279 for 8 mo. at 7%? at $4\frac{1}{2}$%?

23. What is the interest of \$1000 for 6 mo. at 5%? at $5\frac{1}{2}$%? for 1 yr. at $5\frac{1}{2}$%? for 9 mo.?

24. What is the interest of \$240 for 1 yr. 3 mo. at $8\frac{1}{2}$%? for 7 mo.? for $\frac{1}{2}$ yr.?

Ratio — Problems. — 1. What is the relation of $3\frac{1}{2}$ to 4? to 5? to $2\frac{1}{2}$?

2. What is the relation of 5 to 12? to 9? to $2\frac{1}{2}$? to $3\frac{1}{2}$? to $3\frac{1}{5}$?

3. What is the relation of $\frac{1}{2}$ to $\frac{1}{8}$? of $\frac{2}{3}$ to $\frac{4}{5}$? of $\frac{1}{7}$ to $\frac{2}{3}$? of $\frac{1}{8}$ to $\frac{1}{12}$? of $\frac{1}{8}$ to $4\frac{1}{2}$?

4. How many bags, each holding 3 pk., are needed to hold 21 bu. of oats?

5. 7 yd. of ribbon were cut into 3 pieces of equal length. Each piece equaled how many thirds of a yard?

6. At \$1.10 a yard, what part of a yard of cloth can be bought for \$.40?

7. At \$.40 a yard, how much cloth can be bought for \$1.10?

8. If 3 cookies can be bought for 5¢, what is the rate per dozen?

9. When oranges are 40¢ a dozen, how many can be bought for 10¢?

10. Compare $4\frac{1}{3}$ apples with $\frac{2}{3}$ of an apple.

11. If 4 men earn \$$x$ in 30 dy., in what time do 6 men earn an equal amount?

12. If $\frac{7}{8}$ of a farm is worth \$644, how much is the farm worth?

13. How many bushels of coal at \$.10 a bushel can be bought for \$15?

14. The cost of $\frac{3}{8}$ of anything equals what part of the cost of $\frac{1}{2}$ of it?

15. $\frac{1}{3}$ yd. of paving costs \$$x$. What does 1 yd. cost?

16. What is the ratio of the cost of a yard of wire to the cost of 9 in.?

17. If a foot of wire costs \$.12, what is the cost of $\frac{2}{3}$ yd.?

18. What is the cost of 18 bu. of wheat if 15 bu. cost \$7.50?

19. What is the ratio of a rectangle $\frac{1}{2}$ of a by $\frac{1}{2}$ of b to a rectangle a by b?

20. If there are 36 sq. yd. in the square of 3, how many are there in the square of 1?

21. If a surface 9 in. by 4 in. be cut into 4 equal parts, what may be the dimensions of each part?

22. If $\frac{1}{3}$ of the coffee in a equals $\frac{1}{4}$ of the coffee in b, what is the ratio of the coffee in a to the coffee in b?

23. How many cubes $\frac{1}{3}$ yd. long can be cut from a cubic yard?

24. What is the ratio of a 3-in. cube to a 4-in. cube? of a cube 2 in. long to one 4 in. long?

25. What is the ratio of 1 face of a 3-in. cube to 1 face of a 4-in. cube?

26. If the ratio of the length of one square to the length of another is 3, what is the ratio of the larger surface to the smaller? if the ratio is $\frac{3}{4}$?

27. A rug 2 yd. sq. costs \$36. At the same rate per square yard, what is the cost of a rug 3 yd. sq.?

28. How many square feet in one surface of a door 7 ft. high and 3 ft. wide?

29. How many square inches in the surface of an uncovered box 10 in. long, 5 in. wide, and 4 in. high?

30. If the entire surface of a cube is 24 sq. in., what is the distance around it?

31. How many square inches in the surface of a common brick? What is its volume?

32. How many bricks are required to lay a walk 21 ft. long and 1 yd. wide?

33. How many yards of carpet are required for a room 18 ft. long and 6 ft. wide?

34. How many yards of carpet $\frac{3}{4}$ yd. wide are required to carpet a room 5 yd. long and 5 yd. wide?

35. How many square yards in the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 10 ft. high?

36. What would it cost to plaster this room at 25¢ a square yard, making no allowances for openings?

37. If wall paper is 18 in. wide and a roll is 8 yd. long, how many rolls are needed to paper a room 12 ft. long, 9 ft. wide, and 10 ft. high? What is the cost if the paper is 12¢ a roll?

38. What would it cost to plaster one wall of the schoolroom at 22¢ a square yard, making no allowances for openings? to paper it with paper at 9¢ a roll?

39. What, at the same rate, would it cost to plaster the room? to paper both walls and ceiling?

40. The area of a floor is 100 sq. ft. It is 12 ft. long. How wide is it?

41. How many acres in a piece of land 30 rods long and 10 rods wide?

42. In a square piece of ground 40 rods long, how many square rods? How do you know?

43. In 5 acres and 140 sq. rods, how many square rods?

44. At $75 an acre, what is the cost of 40 sq. rods?

45. A man sold $\frac{1}{4}$ section land at $27 an acre and $\frac{1}{2}$ section at $30 an acre. How much did he receive for the land?

46. At $20 an acre, what is the cost of a triangular piece of ground formed by joining the vertices of the opposite angles of a rectangular field 80 rods by 40 rods?

47. A square park is $\frac{1}{4}$ mi. wide. How many acres does it contain?

48. How many acres in a rectangular field 80 rods by 60 rods?

49. How many acres in a triangular field whose base is 80 rods and altitude 60 rods?

50. Compare a solid 1 in. by 1 in. by 3 in. with a 2-in. cube.

51. Compare a 2-in. cube with a solid 1 in. by 1 in. by 8 in.

52. Compare a 3-in. cube with a solid 1 in. by 1 in. by 3 in.

53. A solid whose base is 10 in. sq. and altitude 8 in. equals what part of a 10-in. cube?

54. How many $\frac{1}{4}$ cu. in. blocks can you place in a mortise 6 in. by 6 in. by 6 in.?

55. A room 20 ft. long, 18 ft. wide, and 10 ft. high contains how many cubic feet of air?

56. If there are 48 children in the room, how many cubic feet of air to each child?

57. What is the volume of a solid 8 ft. 4 in. long, 4 ft. 6 in. wide, and 6 ft. high?

58. What is the cost of digging a cellar 24 ft. by 16 ft. by 8 ft. at 30¢ a cubic yard?

59. At $4.50 a cord, what is the cost of a pile of wood 12 ft. long, 12 ft. high, and 8 ft. wide?

60. How many gallons in a rectangular cistern 7 ft. by 3 ft. by 10 ft.?

61. What is the weight of the water in this cistern when full if a cubic foot of water weighs 1000 oz.?

62. How many bushels of oats will fill a bin 6 ft. by

10 ft. by 5 ft.? At 30¢ a bushel, what does a man pay for the oats?

63. How long must a road 33 ft. wide be that its area may equal an acre?

64. What is the ratio of $\frac{1}{12}$ of 1200 to $\frac{1}{12}$ of 756? Then what is the ratio of 1200 to 756?

65. If x equals the number of acres of land which can be bought for \$1200, what equals the number which can be bought for \$756?

66. What is the cost of 6452 paving stones at \$8 per hundred?

67. If 30 lb. of sugar cost \$1.50, what do 175 lb. cost?

68. For 40 horses at \$75 each a man exchanged 125 cattle. At what were the cattle valued? at what per head?

69. A bushel of wheat weighs 60 lb. and a bushel of oats 34 lb. What is the difference in the weight of 10 bu. of each?

70. What is the relative weight of the wheat and the oats in 2 bins of equal size? What is the ratio of 100 bu. of wheat to 100 bu. of oats? of $\frac{5}{8}$ bu. of wheat to $\frac{5}{8}$ bu. of oats?

71. How many boards each 12 ft. long will be needed to build 660 ft. of fencing 4 boards high?

72. A farmer had 47 bu. of apples. He saved 12 bu. for his own use and sold the rest to 5 persons in equal quantities. How many bushels did each person receive?

73. One man spends \$20 a day and another 20¢. What is the ratio of the money they spend in a day? in a month? in a year?

74. John works from 11:45 A.M. until 3:15 P.M. How long does he work?

75. A barrel of flour weighs 196 lb. What does a barrel cost at \$.02 a pound?

76. If a grocer buys flour at $\$.01\frac{1}{2}$ a pound and sells it at \$4 a barrel, what does he gain?

77. If flour is bought at \$3.50 a barrel and sold at \$4, how many barrels must be sold for the dealer to gain \$13?

78. If Fred earns 10¢ a day and spends 6¢, in how many days does he save 20¢?

79. By one pipe 72 gal. per hour run into a cistern and by another 40 gal. per hour run out. If both pipes are left open, how long will the cistern, which holds 160 gal., be in filling?

80. Kate has $\frac{2}{3}$ of $\frac{3}{4}$ of a dollar. She gave Lucy $\frac{1}{2}$ of what she had. What part of a dollar had she left?

81. A boy bought oranges at the rate of 6 for \$.10. For what must he sell them to gain \$.05 on each 3?

82. Lemons that were bought at the rate of 2 for 3¢ were sold at the rate of 3 for 2¢. What was the loss on one?

83. If $\frac{2}{5}$ of the distance you walk on Monday equals $\frac{3}{4}$ of the distance you walk on Tuesday, do you walk the greater distance on Monday or on Tuesday?

84. If $4\frac{2}{3}$ yd. of ribbon cost \$.42, what is the cost of $\frac{2}{3}$ yd.?

85. A gardener receives \$24 for his crop of strawberries, selling them at \$.06 a quart. How many quarts does he sell?

86. If a man pays \$.06 a year for every dollar he borrows, how much does he borrow if he pays \$12?

The relation of \$.06 to \$1 equals the relation of what to \$200?

87. A man and his son receive \$18 for a week's work. The son earns $\frac{1}{2}$ as much as the father. How much does the son earn in 3 wk.?

88. When the son has earned \$27, how much has the father earned?

89. There are 20 quires in 1 ream and 24 sheets in a quire. What is the cost of a ream of paper at $\frac{1}{2}$¢ a sheet?

90. A stationer buys paper at $1.60 a ream. What is the cost of a quire?

91. If he buys at $1.60 a ream and sells at 12¢ a quire, what is his gain?

What is the relation of the selling price of a quire to the cost?

The gain equals what part of the cost? what per cent of it?

92. If he buys at $1.60 a ream and sells at a cent a sheet, what is the relation of the selling price to the cost?

What is the relation of the gain to the cost? the gain equals what per cent of the cost?

93. A merchant sold a shawl for $7 and lost $\frac{1}{8}$ of its value. What part of its value did he receive?

What was the shawl worth?

94. A dealer sells coal so that he gains $\frac{1}{4}$ of the cost. For how many fifths of the cost does he sell it? for what per cent above cost?

95. If he sells it at $6 a ton, what is the relation of $6 to the cost? What is the cost?

96. By selling a table for $\frac{6}{7}$ of its value a merchant gained $1. What was the value?

97. By selling a book for $\frac{5}{6}$ of its cost 20¢ was gained. What was its cost?

The selling price equals what per cent of the cost? The cost equals what per cent of the selling price? Show by drawing.

98. Make and solve problems similar to problem 97.

99. When the relation of the line a to the line b is $\frac{6}{7}$, the difference in their length equals what part of b?

100. A merchant sold flannel for $\frac{11}{10}$ of its cost and gained 10¢ a yard.

What did it cost per yard?

For what did it sell? What per cent was gained?

101. A man spent $\frac{5}{8}$ of his money and had \$100 left. How much had he at first?

102. What is the interest of \$420 for 1 yr. 6 mo. at 6%? at 7%? at $7\frac{1}{2}$%?

103. What is the interest of \$200 for 9 mo. at $5\frac{1}{2}$%?

104. What does a man owe at the end of 7 mo. who borrows \$300 at 8%?

105. A farmer sold sheep for \$81. The ratio of the selling price to the cost was $\frac{9}{7}$. What was the cost?

106. What is the ratio of the amount of work 9 men can do in a day to the amount 8 men can do?

107. If x equals the number of yards of wall 8 men can build in a day, $\frac{5}{8}$ of x equals what?

108. If 9 men can build $\frac{5}{8}$ of a wall in a day, they can build the wall in what part of a day?

109. If 8 men can do a piece of work in 18 days, in what time can 9 men do the work? Why?

1. Draw a 4-in. line and a 6-in. line. What is the longest line that is an exact measure of each?

2. Walter cut 2 pieces of wire, one 36 ft. long and the other 54 ft. long, into equal pieces as long as possible. How many feet long was each piece?

3. A merchant cut 3 pieces of goods, one containing 48 yd., one 72 yd., and one 108 yd., into equal pieces of the greatest possible length. How many yards in each piece?

4. The ratio of a to b is $\frac{2}{3}$. What is the ratio of $\frac{2}{3}$ of b to a? What part of a equals $\frac{1}{3}$ of b? What part of b equals $\frac{1}{2}$ of a?

What is the greatest part of b which is an exact measure of each?

5. If a equals $\frac{3}{4}$ of b, what part of a is an exact measure of both a and b?

What part of b is an exact measure of both?

6. The ratio of x to y is $\frac{4}{5}$. What part of x is an exact measure of both x and y?

What is the ratio of $\frac{1}{4}$ of x to $\frac{1}{5}$ of y?

If $\frac{1}{4}$ of x equals 8, what equals y?

7. The ratio of two rectangles is $\frac{5}{3}$. What part of the smaller is an exact measure of both? what part of the larger?

8. Make and answer similar problems.

9. If $\frac{1}{2}$ of a equals $\frac{1}{4}$ of b, what is the ratio of b to a? of a to b?

10. If $\frac{1}{8}$ of a shorter line equals $\frac{1}{5}$ of a longer, what is the relation of the longer line to the shorter? of the shorter to the longer?

11. If the length of the shorter line is 9 in., what is the length of the longer?

1. $\frac{5}{4}$ is the relation of the line a to the line b. What is the relation of the line b to the line a?

2. $\frac{2}{3}$ is the relation of the rectangle a to the rectangle b. The rectangle b equals what part of the rectangle a?

3. $\frac{1}{8}$ is the relation of the money a man borrowed to the interest he paid. What is the relation of the interest he paid to the money he borrowed?

4. A man borrowed \$300, which equaled $\frac{1}{8}$ of the interest he paid for 1 yr. What interest did he pay? What is the relation of \$18 to \$300?

5. The relation of the money paid for the use of \$200 for 1 yr. to \$200 is .08. How much was paid?

.08 is the relation of \$16 to the amount a man borrowed. He borrowed how many eighths of \$16? How much did he borrow?

6. The relation of the money Mr. Smith paid for the use of \$400 for 1 yr. to \$400 is .06. How much did he pay for its use?

What is the relation of the money he borrowed to the money he paid for its use for 1 yr. ?

7. A man borrowed $200 for 1 yr. and paid $12 for its use. The money he paid for its use was equal to what part of the money he borrowed ?

What is the ratio of $\frac{1}{2}$ of 12 to $\frac{1}{2}$ of 200 ? What, then, is the ratio of 12 to 200 ?

8. A man borrowed $200 for 1 yr. and paid $14 interest. What was the ratio of the interest paid to the money borrowed ?

If he pays $16 for the use of $200 for 1 yr., the interest equals how many hundredths of the money borrowed ?

9. Mr. Jones borrows $480 of Mr. Brown and agrees to pay him for its use an amount equal to .1 of what he borrows. What amount will Mr. Jones owe at the end of the year ?

10. For every year that a man uses $300 he pays the lender $12. At the end of a certain time he pays $60. How long did he keep the $300 ?

1. A woman sells eggs to a grocer at 16¢ a dozen, which is $\frac{4}{5}$ of the price the grocer sells them at. What does he get for a dozen ?

His profit equals what part of the cost ?

For what per cent of the cost does he sell them ?

2. By selling butter at $8\frac{1}{3}$¢ a pound a woman gets but $\frac{5}{6}$ of the market price. What is the ratio of the market price to $\frac{5}{6}$ of the market price ? What is the market price ?

3. A dealer sold lard at $6\frac{1}{4}$¢ a pound and by so doing lost $\frac{1}{5}$ of what he paid for it. What was its cost ?

4. A man bought a horse for $116 and sold it so as to gain $12\frac{1}{2}$¢ on every dollar the horse cost him. The selling price of the horse equaled what per cent of the cost ?

5. A grocer bought $100 worth of apples and sold them for $140. What part of the capital invested did he lose? how many cents on each dollar?

6. A merchant asked $2 a yard for broadcloth, which equaled $\frac{4}{5}$ of the cost. What was the cost?

If he sold it for $\frac{3}{4}$ of what he asked for it, did he gain or lose?

7. A milliner bought hats at $1 each and sold them for 1.66\frac{2}{3}$ each. The gain equaled what part of the money invested?

8. A man invested $1400 in land and sold it at a profit of 12$\frac{1}{2}$¢ on every dollar. What was his profit? For how much did he sell the land?

The ratio of the gain to $1400 equals the ratio of 12$\frac{1}{2}$¢ to what?

The ratio of the selling price to $1400 equals the ratio of what?

9. What will 500 acres of government land cost at 1.83\frac{1}{3}$ an acre? at 1.87\frac{1}{2}$?

10. Carpeting which cost $1 a yard was sold for 87$\frac{1}{2}$¢ a yard. The loss equaled what part of the cost?

11. A merchant bought shoes at $1 a pair and sold them for $1.75 a pair. His gain equaled what part of the cost?

12. A furniture dealer paid $100 for chairs at $1 each. He sold them for $\frac{4}{5}$ more than they cost him. What was the selling price of each chair? What did he receive for all?

13. A milliner bought feathers at $1 and marked them to sell for $\frac{1}{4}$ more than they cost. What was the marked price?

She sold them for $\frac{1}{5}$ less than the marked price. What did she get for them?

Did she make or lose by buying and selling the feathers?

14. If the interest on a certain sum of money for 1 yr. is $60, what is it for 1 yr. 9 mo.?

15. At 40¢ a pound, what will 5 lb. 12 oz. of butter cost?

16. What will 3 pk. of plums cost at $2 a bushel?

17. What will 4 pk. 7 qt. of cherries cost at $1 a peck?

18. A grocer sold beans at 8¢ per quart, which was $\frac{1}{8}$ more than he paid for them. What did they cost per bushel?

19. A merchant sold a dress pattern of 8 yd. at $1.50 per yard, receiving $\frac{1}{3}$ more than the goods cost him. What was his profit?

20. A gardener received $4.80 for a crate of 24 boxes of berries. To one customer he sold 6 boxes, to another 8, to another 5, to another 3, and to another 4. How much money did he receive from each?

21. The margin of a printed page is $\frac{2}{5}$ of its surface. The printed surface is 4 in. by 5 in. How many square inches in the page?

In a book of 300 such pages, how many square inches of paper?

22. The matting around a picture is 4 in. deep. The glass over the picture is 16 in. by 18 in. How many square inches in the surface of the picture, not including the matting?

23. How many feet of moulding 3 in. wide will be required to frame the picture?

24. If 75¢ pays for $\frac{3}{8}$ yd. of cloth, how many yards will $7 buy? $3\frac{1}{2}$? $2\frac{3}{4}$? $5\frac{1}{4}$?

25. A cistern 6 ft. in diameter holds 6.71 bbl. of water per foot. How many barrels in the cistern when the water is 10 ft. deep? 8 ft.? 5 ft.? 3 ft.? 20 ft.?

26. A wagon-bed 10 ft. long, 3 ft. wide, and 18 in. deep holds 36 bu. of corn. If its width be increased 3 in., how many bushels will it hold? Had its height been increased 3 in. and its width remained 3 ft., how many bushels would it have held?